Lecture 10: Heterogeneous Agents

Jacob Adenbaum

University of Edinburgh

Spring 2024

Where we've been

- So far, we've learned how to write economic models **recursively**.
- Our prototypical example was the Neoclassical Growth Model:

$$\begin{aligned} v(k) = \max_{c,k'} \quad & u(c) + \beta v(k') \\ \text{s.t.} \quad & c + k' \leq F(k) + (1-\delta)k \end{aligned}$$

- Once they're written recursively, we've learned how to **solve** them (find a function that satisfies the recursive relationship)
- Once they're solved, we learned how to **simulate** them, and use the simulated data to **estimate** parameters
- With representative agents, an equilibrium in these models is not very complicated
  - If firms rent capital from household, we get $r = F'(k)$, etc...

Where we've been

- So far, we've learned how to write economic models **recursively**.
- Our prototypical example was the Neoclassical Growth Model:

$$\begin{aligned} v(k) = \max_{c,k'} \quad & u(c) + \beta v(k') \\ \text{s.t.} \quad & c + k' \leq F(k) + (1-\delta)k \end{aligned}$$

- Once they're written recursively, we've learned how to **solve** them (find a function that satisfies the recursive relationship)
- Once they're solved, we learned how to **simulate** them, and use the simulated data to **estimate** parameters
- With representative agents, an equilibrium in these models is not very complicated
  - If firms rent capital from household, we get $r = F'(k)$, etc...

Where we've been

- So far, we've learned how to write economic models **recursively**.
- Our prototypical example was the Neoclassical Growth Model:

$$
\begin{aligned}
v(k) = \max_{c,k'} \quad & u(c) + \beta v(k') \\
\text{s.t.} \quad & c + k' \leq F(k) + (1-\delta)k
\end{aligned}
$$

- Once they're written recursively, we've learned how to **solve** them (find a function that satisfies the recursive relationship)
- Once they're solved, we learned how to **simulate** them, and use the simulated data to **estimate** parameters
- With representative agents, an equilibrium in these models is not very complicated
  - If firms rent capital from household, we get $r = F'(k)$, etc...

Where we're going

- Computing with General Equilibrium
  - Many interesting models do not feature a representative household
  - When there are many heterogeneous agents in our models, there are several special concerns
  - Mostly about how we compute the **market clearing prices**
  - How do we approach these models computationally?
- Policy Analysis
  - The models we've worked on so far have all been efficient (Limited role for policy)
  - It's hard to even think about redistribution in a model with just a representative household
  - In many models, the government can step in to correct market failures, but we need to know: what is the optimal policy?

Where we're going

- Computing with General Equilibrium
  - Many interesting models do not feature a representative household
  - When there are many heterogeneous agents in our models, there are several special concerns
  - Mostly about how we compute the **market clearing prices**
  - How do we approach these models computationally?
- Policy Analysis
  - The models we've worked on so far have all been efficient (Limited role for policy)
  - It's hard to even think about redistribution in a model with just a representative household
  - In many models, the government can step in to correct market failures, but we need to know: what is the optimal policy?

Where we're going

- Computing with General Equilibrium
  - Many interesting models do not feature a representative household
  - When there are many heterogeneous agents in our models, there are several special concerns
  - Mostly about how we compute the **market clearing prices**
  - How do we approach these models computationally?
- Policy Analysis
  - The models we've worked on so far have all been efficient (Limited role for policy)
  - It's hard to even think about redistribution in a model with just a representative household
  - In many models, the government can step in to correct market failures, but we need to know: what is the **optimal policy**?

Where we're going

- Computing with General Equilibrium
  - Many interesting models do not feature a representative household
  - When there are many heterogeneous agents in our models, there are several special concerns
  - Mostly about how we compute the **market clearing prices**
  - How do we approach these models computationally?
- Policy Analysis
  - The models we've worked on so far have all been efficient (Limited role for policy)
  - It's hard to even think about redistribution in a model with just a representative household
  - In many models, the government can step in to correct market failures, but we need to know: what is the **optimal policy**?

Section 1

Heterogeneous Agent Models: Aiyagari (1994, QJE)

Aiyagari (1994, QJE): Prototypical Heterogeneous Agent Model

$$\begin{aligned} v(a,y) = \max_{c,a' \geq 0} \quad & u(c) + \beta \mathbb{E}\left[v(a',y')|y\right] \\ \text{s.t.} \quad & c + a' \leq (1+r)a + y \\ & \log(y') = \rho \log(y) + \epsilon \\ & \epsilon \sim N(0,\sigma) \end{aligned} \tag{1}$$

- Consider the problem of a large group of households who must save for the future
- They are heterogeneous in their current income y, and in their level of assets a.
  - Log income follows an AR(1) process
  - Labor supplied inelastically (no choice of how much to work)
- Derive flow utility $u(c)$ from consumption, and discount the future at rate β
- Can save for the future at a rate $1+r$, but cannot borrow.
  - **Markets are incomplete** (There are certain risks that they cannot insure against)
- So far, this should look very familiar from your problem set...

Aiyagari (1994, QJE): Prototypical Heterogeneous Agent Model

$$
\begin{aligned}
v(a, y) = \max_{c, a' \geq 0} \quad & u(c) + \beta \mathbb{E}\left[v(a', y') | y\right] \\
\text{s.t.} \quad & c + a' \leq (1+r)a + y \\
& \log(y') = \rho \log(y) + \epsilon \\
& \epsilon \sim N(0, \sigma)
\end{aligned}
\tag{1}
$$

- Consider the problem of a large group of households who must save for the future
- They are heterogeneous in their current income y, and in their level of assets a.
  - Log income follows an AR(1) process
  - Labor supplied inelastically (no choice of how much to work)
- Derive flow utility $u(c)$ from consumption, and discount the future at rate β
- Can save for the future at a rate $1 + r$, but cannot borrow.
  - **Markets are incomplete** (There are certain risks that they cannot insure against)
- So far, this should look very familiar from your problem set...

Aiyagari (1994, QJE): Prototypical Heterogeneous Agent Model

$$
\begin{aligned}
v(a,y) = \max_{c,a' \geq 0} \quad & u(c) + \beta \mathbb{E}\left[v(a',y')|y\right] \\
\text{s.t.} \quad & c + a' \leq (1+r)a + y \\
& \log(y') = \rho \log(y) + \epsilon \\
& \epsilon \sim N(0,\sigma)
\end{aligned} \tag{1}
$$

- Consider the problem of a large group of households who must save for the future
- They are heterogeneous in their current income y, and in their level of assets a.
  - Log income follows an AR(1) process
  - Labor supplied inelastically (no choice of how much to work)
- Derive flow utility $u(c)$ from consumption, and discount the future at rate β
- Can save for the future at a rate $1 + r$, but cannot borrow.
  - **Markets are incomplete** (There are certain risks that they cannot insure against)
- So far, this should look very familiar from your problem set...

Extending Bewley to Aiyagari

- If we take r as given, and just consider the households' consumption savings problem, then we know how to solve
  - We saw that it's not much more complicated than the neoclassical growth model with stochastic productivity
  - But r is a price: we want it to be set, in equilibrium, to clear the market for assets
- Supply Side: Suppose we have a representative firm, with production function $F(k)$, who rents capital from the households at a price r.

$$\max_k F(k) - rk \implies F'(k) = r \implies k = K(r) \tag{2}$$

For some function $K(r)$. If $F(k) = k^\alpha$, then $K(r) = \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}$

- Distribution of agents: let $\Lambda(a, y)$ be the cumulative distribution function of assets and income in the economy (with pdf λ)

Extending Bewley to Aiyagari

- If we take r as given, and just consider the households' consumption savings problem, then we know how to solve
  - We saw that it's not much more complicated than the neoclassical growth model with stochastic productivity
  - But r is a price: we want it to be set, in equilibrium, to clear the market for assets
- Supply Side: Suppose we have a representative firm, with production function $F(k)$, who rents capital from the households at a price r.

$$\max_k F(k) - rk \implies F'(k) = r \implies k = K(r) \tag{2}$$

For some function $K(r)$. If $F(k) = k^\alpha$, then $K(r) = \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}$

- Distribution of agents: let $\Lambda(a, y)$ be the cumulative distribution function of assets and income in the economy (with pdf λ)

Equilibrium

- A **recursive stationary equilibrium** in this model is a set of
    1. Consumption and savings policy functions $g_c(a, y)$ and $g_a(a, y)$,
    2. An interest rate r, and
    3. A distribution $\Lambda(a, y)$ over assets and income levels
- Such that:
    1. **Optimality:** g_c and g_a solve the household's consumption/savings problem, given r
    2. **Market Clearing:** The interest rate r clears the market for capital

$$K(r) = \int a \, d\Lambda(a, y) = \int \int a \, \lambda(a, y) \, da \, dy \tag{3}$$

    3. **Stationarity:** Given the policy functions g_c and g_a, and the interest rate r, the distribution Λ is unchanging over time
- We know what optimality means – need to solve the household's dynamic program as we have been doing
- Need to spend a little bit of time thinking through market clearing and stationarity

Equilibrium

- A **recursive stationary equilibrium** in this model is a set of
  1. Consumption and savings policy functions $g_c(a, y)$ and $g_a(a, y)$,
  2. An interest rate r, and
  3. A distribution $\Lambda(a, y)$ over assets and income levels
- Such that:
  1. **Optimality**: g_c and g_a solve the household's consumption/savings problem, given r
  2. **Market Clearing:** The interest rate r clears the market for capital

$$K(r) = \int a \, d\Lambda(a, y) = \int \int a \, \lambda(a, y) \, da \, dy \tag{3}$$

  3. **Stationarity:** Given the policy functions g_c and g_a, and the interest rate r, the distribution Λ is unchanging over time
- We know what optimality means – need to solve the household's dynamic program as we have been doing
- Need to spend a little bit of time thinking through market clearing and stationarity

Equilibrium

- A **recursive stationary equilibrium** in this model is a set of
  1. Consumption and savings policy functions $g_c(a, y)$ and $g_a(a, y)$,
  2. An interest rate r, and
  3. A distribution $\Lambda(a, y)$ over assets and income levels
- Such that:
  1. **Optimality**: g_c and g_a solve the household's consumption/savings problem, given r
  2. **Market Clearing:** The interest rate r clears the market for capital

$$K(r) = \int a \, d\Lambda(a, y) = \int \int a \, \lambda(a, y) \, da \, dy \quad (3)$$

  3. **Stationarity:** Given the policy functions g_c and g_a, and the interest rate r, the distribution Λ is unchanging over time
- We know what optimality means – need to solve the household's dynamic program as we have been doing
- Need to spend a little bit of time thinking through market clearing and stationarity

Equilibrium

- A **recursive stationary equilibrium** in this model is a set of
  1. Consumption and savings policy functions $g_c(a, y)$ and $g_a(a, y)$,
  2. An interest rate r, and
  3. A distribution $\Lambda(a, y)$ over assets and income levels
- Such that:
  1. **Optimality**: g_c and g_a solve the household's consumption/savings problem, given r
  2. **Market Clearing:** The interest rate r clears the market for capital
  $$K(r) = \int a \, d\Lambda(a, y) = \int \int a \, \lambda(a, y) \, da \, dy \tag{3}$$
  3. **Stationarity:** Given the policy functions g_c and g_a, and the interest rate r, the distribution Λ is unchanging over time
- We know what optimality means – need to solve the household's dynamic program as we have been doing
- Need to spend a little bit of time thinking through market clearing and stationarity

Equilibrium

- A **recursive stationary equilibrium** in this model is a set of
  1. Consumption and savings policy functions $g_c(a, y)$ and $g_a(a, y)$,
  2. An interest rate r, and
  3. A distribution $\Lambda(a, y)$ over assets and income levels
- Such that:
  1. **Optimality**: g_c and g_a solve the household's consumption/savings problem, given r
  2. **Market Clearing:** The interest rate r clears the market for capital

$$K(r) = \int a \, d\Lambda(a, y) = \int \int a \, \lambda(a, y) \, da \, dy \tag{3}$$

  3. **Stationarity:** Given the policy functions g_c and g_a, and the interest rate r, the distribution Λ is unchanging over time
- We know what optimality means – need to solve the household's dynamic program as we have been doing
- Need to spend a little bit of time thinking through market clearing and stationarity

Stationarity

- A stationary distribution is one that is not changing over time
- If we step the distribution forward one time period, using our policy rules, we should get the same distribution back out again
- Let $\pi(y'|y)$ denote the conditional pdf of income tomorrow given that income today is y.
- Then we can write the **law of motion** for Λ as
$$\Lambda(a, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{y} \int_{0}^{\infty} \mathbb{1}\left\{g_a(a_0, y_0) \leq a'\right\} \pi(y'|y_0)\, \lambda(a_0, y_0)\, da_0\, dy'\, d_{y_0}$$
- This is just fancy math for: if I step my simulated distribution of agents forward one period, the overall distribution should not change
- Each agent is moving around through the distribution, but on average it stays the same
- In this class, we will never compute those integrals directly – we will always be approximating the distribution using a simulated set with a discrete number of agents

Stationarity

- A stationary distribution is one that is not changing over time
- If we step the distribution forward one time period, using our policy rules, we should get the same distribution back out again
- Let $\pi(y'|y)$ denote the conditional pdf of income tomorrow given that income today is y.
- Then we can write the **law of motion** for Λ as
$$\Lambda(a, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{y} \int_{0}^{\infty} \mathbb{1}\left\{g_a(a_0, y_0) \leq a'\right\} \pi(y'|y_0) \, \lambda(a_0, y_0) \, da_0 \, dy' \, d_{y_0}$$
- This is just fancy math for: if I step my simulated distribution of agents forward one period, the overall distribution should not change
- Each agent is moving around through the distribution, but on average it stays the same
- In this class, we will never compute those integrals directly – we will always be approximating the distribution using a simulated set with a discrete number of agents

Stationarity

- A stationary distribution is one that is not changing over time
- If we step the distribution forward one time period, using our policy rules, we should get the same distribution back out again
- Let $\pi(y'|y)$ denote the conditional pdf of income tomorrow given that income today is y.
- Then we can write the **law of motion** for Λ as

$$\Lambda(a, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{y} \int_{0}^{\infty} \mathbb{1}\left\{g_a(a_0, y_0) \leq a'\right\}\ \pi(y'|y_0)\ \lambda(a_0, y_0)\ da_0\ dy'\ d_{y_0}$$

- This is just fancy math for: if I step my simulated distribution of agents forward one period, the overall distribution should not change
- Each agent is moving around through the distribution, but on average it stays the same
- In this class, we will never compute those integrals directly – we will always be approximating the distribution using a simulated set with a discrete number of agents

Stationarity

- A stationary distribution is one that is not changing over time
- If we step the distribution forward one time period, using our policy rules, we should get the same distribution back out again
- Let $\pi(y'|y)$ denote the conditional pdf of income tomorrow given that income today is y.
- Then we can write the **law of motion** for Λ as

$$\Lambda(a, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{y} \int_{0}^{\infty} \mathbb{1}\left\{g_a(a_0, y_0) \leq a'\right\} \, \pi(y'|y_0) \, \lambda(a_0, y_0) \, da_0 \, dy' \, d_{y_0}$$

- This is just fancy math for: if I step my simulated distribution of agents forward one period, the overall distribution should not change
- Each agent is moving around through the distribution, but on average it stays the same
- In this class, we will never compute those integrals directly – we will always be approximating the distribution using a simulated set with a discrete number of agents

Stationarity

- A stationary distribution is one that is not changing over time
- If we step the distribution forward one time period, using our policy rules, we should get the same distribution back out again
- Let $\pi(y'|y)$ denote the conditional pdf of income tomorrow given that income today is y.
- Then we can write the **law of motion** for Λ as

$$\Lambda(a, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{y} \int_{0}^{\infty} \mathbb{1}\left\{g_a(a_0, y_0) \leq a'\right\} \pi(y'|y_0) \lambda(a_0, y_0)\, da_0\, dy'\, d_{y_0}$$

- This is just fancy math for: if I step my simulated distribution of agents forward one period, the overall distribution should not change
- Each agent is moving around through the distribution, but on average it stays the same
- In this class, we will never compute those integrals directly – we will always be approximating the distribution using a simulated set with a discrete number of agents

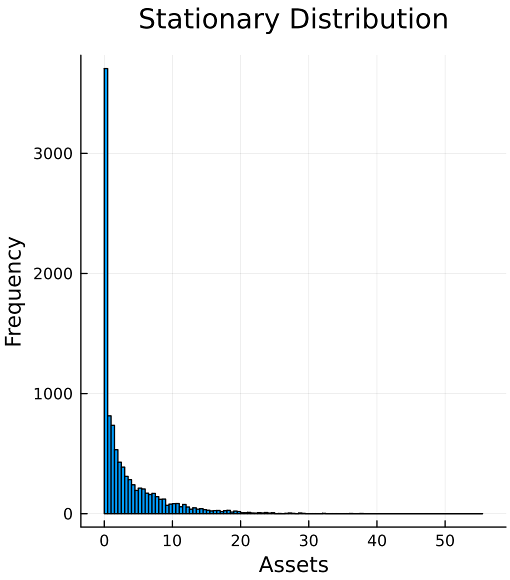
Stationarity

- In general, your distributions will usually converge to a single, stationary distribution
  As long as it's possible to move from every point in the state space to every other point in the state space (full mixing)
- We say that the distribution has converged if the histogram of assets and income has stopped changing
- The thing we actually want is to calculate the total assets in the economy:

$$A(\Lambda) = \int \int a\lambda(a, y)\; da\; dy$$

- Take the average of the assets of our agents in our simulated distribution:

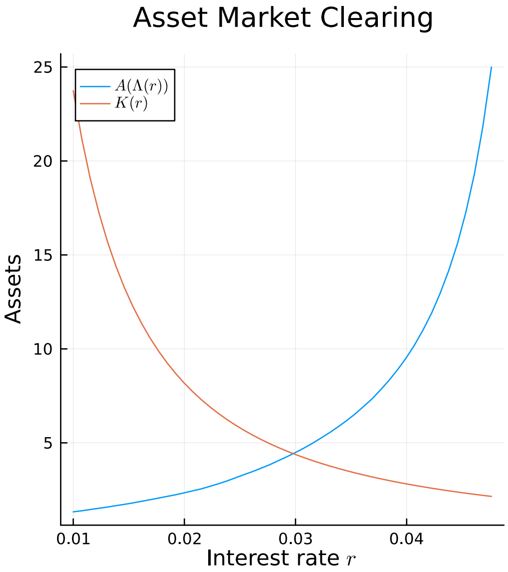
$$A(\Lambda) \approx \frac{1}{N} \sum_{i=1}^{N} a_i$$



Market Clearing

- Remember that our stationary distribution is calculated using policy functions g_c and g_a that take r as given.
- That means we can really write $\Lambda(r)$: the stationary distribution of assets depends on the interest rate
- Our simulation results also depend on r: average assets are $A(\Lambda(r))$
- Supply and Demand:
  - $A(\Lambda(r))$ is our upward sloping supply curve of assets
  - $K(r)$ is our downward sloping demand curve for capital
  - The market clearing price is the r that sets

$$K(r) = A(\Lambda(r))$$

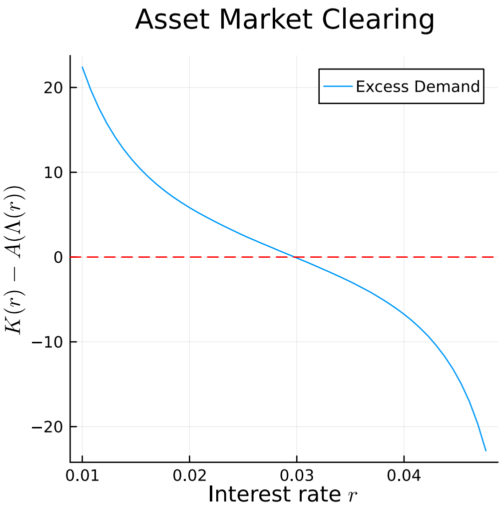


Market Clearing: Root finding approach

- Recast problem as root finding on excess demand:

$$ED(r) = K(r) - A(\Lambda(r)) \qquad (4)$$

- With a sensible root finding procedure, you will typically converge within 10 iterations for a 1D problem
- If you have multiple markets to clear, then it's a multivariate root finding problem – harder to do
- Be careful of tolerances in your root finding procedure Simulations are noisy, and so you may not be able to solve your root finding problem accurately beyond a tolerance of 10^{-3} without a prohibitively large computational cost



There are more clever approaches to simulating the distribution of assets, but they tend to be less intuitive

Aiyagari: Wrapping Up

- In general, our computational tools allow us to analyze these types of heterogenous agent problems
- When we do, we will have to think more carefully about how to deal with market clearing and other equilibrium conditions
- Very few limits (other than computational cost) on which dynamic models we can solve
- Especially when you move into the world of models with many agents, and nontrivial dispersion in wealth/human capital/income/etc..., these models are *not* amenable to being solved on pen and paper
- For many problems, VFI is the slowest, but most robust solution
  - There are other approaches, but they all tend to be more situational (although they often obtain large speed gains)
  - There are approaches (like policy function iteration, and others) that can speed up VFI
- Oftentimes, without a smarter approach, the majority of your time will be spent in the simulation code, rather than solving the model

Aiyagari: Wrapping Up

- In general, our computational tools allow us to analyze these types of heterogenous agent problems
- When we do, we will have to think more carefully about how to deal with market clearing and other equilibrium conditions
- Very few limits (other than computational cost) on which dynamic models we can solve
- Especially when you move into the world of models with many agents, and nontrivial dispersion in wealth/human capital/income/etc..., these models are *not* amenable to being solved on pen and paper
- For many problems, VFI is the slowest, but most robust solution
  - There are other approaches, but they all tend to be more situational (although they often obtain large speed gains)
  - There are approaches (like policy function iteration, and others) that can speed up VFI
- Oftentimes, without a smarter approach, the majority of your time will be spent in the simulation code, rather than solving the model

Aiyagari: Wrapping Up

- In general, our computational tools allow us to analyze these types of heterogenous agent problems
- When we do, we will have to think more carefully about how to deal with market clearing and other equilibrium conditions
- Very few limits (other than computational cost) on which dynamic models we can solve
- Especially when you move into the world of models with many agents, and nontrivial dispersion in wealth/human capital/income/etc..., these models are *not* amenable to being solved on pen and paper
- For many problems, VFI is the slowest, but most robust solution
  - There are other approaches, but they all tend to be more situational (although they often obtain large speed gains)
  - There are approaches (like policy function iteration, and others) that can speed up VFI
- Oftentimes, without a smarter approach, the majority of your time will be spent in the simulation code, rather than solving the model

Section 2

Policy Experiments

Why do we need a model for policy evaluation?

Predictive accuracy

There are many cases where the reduced-form elasticities you get from running a regression (even a well-identified regression) are *not* good predictors of how people will behave if you make changes to policy

- People who are forward-looking are much more responsive to permanent changes than temporary changes

  You have to be careful about which elasticities you're actually measuring

- People can respond to changes in policy in unexpected ways

  E.g. Changes in inflation expectations in the 1970s

- Predictions that are not grounded in a model of people's underlying choices are vulnerable to the Lucas Critique: behavior rules estimated in the data are not invariant to policy

- Making sense of the data available: Indirect Inference

- Often, economic models have good predictions, even out of sample.

Why do we need a model for policy evaluation?

Predictive accuracy

There are many cases where the reduced-form elasticities you get from running a regression (even a well-identified regression) are *not* good predictors of how people will behave if you make changes to policy

- People who are forward-looking are much more responsive to permanent changes than temporary changes

  You have to be careful about which elasticities you're actually measuring

- People can respond to changes in policy in unexpected ways

  E.g. Changes in inflation expectations in the 1970s

- Predictions that are not grounded in a model of people's underlying choices are vulnerable to the Lucas Critique: behavior rules estimated in the data are not invariant to policy

- Making sense of the data available: Indirect Inference

- Often, economic models have good predictions, even out of sample.

Why do we need a model for policy evaluation?

Predictive accuracy

There are many cases where the reduced-form elasticities you get from running a regression (even a well-identified regression) are *not* good predictors of how people will behave if you make changes to policy

- People who are forward-looking are much more responsive to permanent changes than temporary changes

  You have to be careful about which elasticities you're actually measuring

- People can respond to changes in policy in unexpected ways

  E.g. Changes in inflation expectations in the 1970s

- Predictions that are not grounded in a model of people's underlying choices are vulnerable to the Lucas Critique: behavior rules estimated in the data are not invariant to policy

- Making sense of the data available: Indirect Inference

- Often, economic models have good predictions, even out of sample.

Why do we need a model for policy evaluation?

Predictive accuracy

There are many cases where the reduced-form elasticities you get from running a regression (even a well-identified regression) are *not* good predictors of how people will behave if you make changes to policy

- People who are forward-looking are much more responsive to permanent changes than temporary changes

  You have to be careful about which elasticities you're actually measuring

- People can respond to changes in policy in unexpected ways

  E.g. Changes in inflation expectations in the 1970s

- Predictions that are not grounded in a model of people's underlying choices are vulnerable to the Lucas Critique: behavior rules estimated in the data are not invariant to policy

- Making sense of the data available: Indirect Inference

- Often, economic models have good predictions, even out of sample.

Why do we need a model for policy evaluation?

Predictive accuracy

There are many cases where the reduced-form elasticities you get from running a regression (even a well-identified regression) are *not* good predictors of how people will behave if you make changes to policy

- People who are forward-looking are much more responsive to permanent changes than temporary changes

  You have to be careful about which elasticities you're actually measuring

- People can respond to changes in policy in unexpected ways

  E.g. Changes in inflation expectations in the 1970s

- Predictions that are not grounded in a model of people's underlying choices are vulnerable to the Lucas Critique: behavior rules estimated in the data are not invariant to policy

- Making sense of the data available: Indirect Inference

- Often, economic models have good predictions, even out of sample.

Why do we need a model for policy evaluation?

Counterfactuals and Welfare Analysis

- Counterfactuals are at the heart of the questions we want to answer:
    - How will people's behavior in response to a policy that has never been implemented?
    - How would they have behaved if we hadn't implemented some policy?
- In nontrivial models, we need a model in order to evaluate the welfare impacts of a change in policy
    - Will people be better off on average after a tax reform?
    - By how much?
    - Will this reform increase or decrease inequality?
    - How are the gains distributed?
- Without a model, you cannot hope to answer these kinds of questions

Tax Reform in Aiyagari

- In Aiyagari models, generally people tend to over-save relative to what the social planner would choose
  - Fear of a sequence of very many negative shocks
  - If you hit your borrowing constraint, you may wind up with very low consumption
  - Strong precautionary motive for savings, at the individual level, to self-insure against income risk
- Suppose that the government imposes a tax on capital income τ, and redistributes the money with a lump sum tax T (let's say, $\tau = 30\%$)
- How can we model the effects of this tax reform?

Tax Reform in Aiyagari

- In Aiyagari models, generally people tend to over-save relative to what the social planner would choose
  - Fear of a sequence of very many negative shocks
  - If you hit your borrowing constraint, you may wind up with very low consumption
  - Strong precautionary motive for savings, at the individual level, to self-insure against income risk
- Suppose that the government imposes a tax on capital income τ, and redistributes the money with a lump sum tax T (let's say, $\tau = 30\%$)
- How can we model the effects of this tax reform?

Tax Reform in Aiyagari: Updated Model

$$
\begin{aligned}
v(a, y) = \max_{c, a' \geq 0} \quad & u(c) + \beta \mathbb{E}\left[v(a', y') | y\right] \\
\text{s.t.} \quad & c + a' \leq \left[1 + r(1-\tau)\right] a + y + T \\
& \log(y') = \rho \log(y) + \epsilon \\
& \epsilon \sim N(0, \sigma)
\end{aligned} \tag{5}
$$

- Only change to the Bellman equations are in the budget constraint: consumers take τ and T as given
- New considerations:
  - Taxes distort savings behavior $\implies$ different r in equilibrium
  - Government needs to balance its budget $\implies$ find T such that

$$\int \int \tau r a \, \lambda(a, y) \, da \, dy = T$$

  - Both of these will change consumer behavior – we have to solve for all of them jointly

Tax Reform in Aiyagari: Solution Algorithm

Fix $\tau = 30\%$. Treat these market clearing conditions as nested problems:

- Define $D(r, T)$ to be the government's budget deficit
- Define $ED(r, T)$ to be the excess demand for capital
- Algorithm:
  1. For any given r, solve for the T that balances the government's budget (solving and simulating the model). That is, solve the root finding problem
  $$D(r, T) = 0$$
  as a function of T, holding r fixed. Call the results $T^\star(r)$
  2. Solve the root finding problem for
  $$ED(r, T^\star(r)) = 0$$
  Call the result $r^\star$
- Our final (r, T) are $(r^\star, T^\star(r^\star))$.

We'll go over code for how to do this in tutorial

Tax Reform in Aiyagari: Solution Algorithm

Fix $\tau = 30\%$. Treat these market clearing conditions as nested problems:

- Define $D(r, T)$ to be the government's budget deficit
- Define $ED(r, T)$ to be the excess demand for capital
- Algorithm:
  1. For any given r, solve for the T that balances the government's budget (solving and simulating the model). That is, solve the root finding problem

     $$D(r, T) = 0$$

     as a function of T, holding r fixed. Call the results $T^\star(r)$
  2. Solve the root finding problem for

     $$ED(r, T^\star(r)) = 0$$

     Call the result $r^\star$
- Our final (r, T) are $(r^\star, T^\star(r^\star))$.

We'll go over code for how to do this in tutorial

Tax Reform in Aiyagari: Solution Algorithm

Fix $\tau = 30\%$. Treat these market clearing conditions as nested problems:

- Define $D(r, T)$ to be the government's budget deficit
- Define $ED(r, T)$ to be the excess demand for capital
- Algorithm:
  1. For any given r, solve for the T that balances the government's budget (solving and simulating the model). That is, solve the root finding problem

     $$D(r, T) = 0$$

     as a function of T, holding r fixed. Call the results $T^\star(r)$
  2. Solve the root finding problem for

     $$ED(r, T^\star(r)) = 0$$

     Call the result $r^\star$
- Our final (r, T) are $(r^\star, T^\star(r^\star))$.

We'll go over code for how to do this in tutorial

Tax Reform in Aiyagari: Solution Algorithm

Fix $\tau = 30\%$. Treat these market clearing conditions as nested problems:

- Define $D(r, T)$ to be the government's budget deficit
- Define $ED(r, T)$ to be the excess demand for capital
- Algorithm:
  1. For any given r, solve for the T that balances the government's budget (solving and simulating the model). That is, solve the root finding problem
  $$D(r, T) = 0$$
  as a function of T, holding r fixed. Call the results $T^{\star}(r)$
  2. Solve the root finding problem for
  $$ED(r, T^{\star}(r)) = 0$$
  Call the result $r^{\star}$
- Our final (r, T) are $(r^{\star}, T^{\star}(r^{\star}))$.

We'll go over code for how to do this in tutorial

Tax Reform in Aiyagari: Solution Algorithm

Fix $\tau = 30\%$. Treat these market clearing conditions as nested problems:

- Define $D(r, T)$ to be the government's budget deficit
- Define $ED(r, T)$ to be the excess demand for capital
- Algorithm:
  1. For any given r, solve for the T that balances the government's budget (solving and simulating the model). That is, solve the root finding problem
     $$D(r, T) = 0$$
     as a function of T, holding r fixed. Call the results $T^\star(r)$
  2. Solve the root finding problem for
     $$ED(r, T^\star(r)) = 0$$
     Call the result $r^\star$
- Our final (r, T) are $(r^\star, T^\star(r^\star))$.

We'll go over code for how to do this in tutorial