Consumption

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Slides adapted from Jonna Olsson

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Roadmap

Where we've been

- Developed a theory of production, and figured out what it means for aggregate prices
- Moved on to the demand side of the economy, and figured out the determinants of the optimal capital stock

$$\underbrace{\frac{MPK_{t+1}}{1+\mu}}_{\text{Increased Revenue}} = \underbrace{r_{t+1} + \delta}_{\text{User cost of capital}}$$

- We showed that (in the Cobb-Douglas case) our investment function depends on three factors: $I(r, Y^e, K)$

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- Now we move on to the other side of aggregate demand: consumption
- We will analyze the determinants of household consumption behavior
- What motivates a household to save for the future
- How do their choices depend on macroeconomic conditions?

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The user side of GDP

Remember from Week 1:

$$Y = C + I + C^G + I^G + X - IM$$

In our simplified model, we are removing (for now):

- Rest of the world
- Government

Thus, in our simplified model, aggregate demand is the sum of

- Private consumption (made by households)
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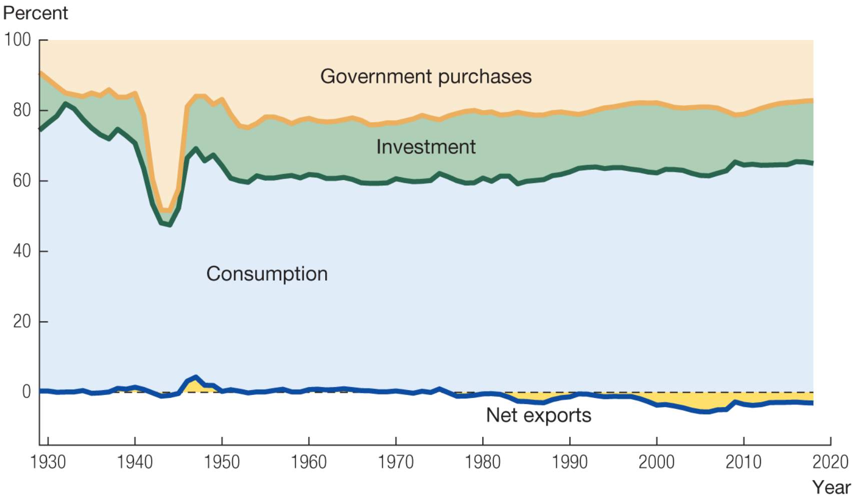

Figure: User side of GDP for the U.S. Source: Jones (2020)

Consumption

- **Two reasons why so much time is spent on studying consumption:**
  - Consumption is the largest component of aggregate demand
  - It is by consumption welfare is created
- **To model consumption involves modelling choices:**
  - Consumption of good x or good y
  - Consumption of a bundle of goods or consumption of leisure
  - Consumption of goods today or consumption of goods later
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- How much does a two-period consumer consume in each period?
- We need to figure out:
  1. what the consumer can afford
  2. what the consumer wants to do, given what she can afford
- Let us carefully specify the problem. We assume that the consumer:
  - Lives for two periods: period 1 and 2
  - Receives real labor income in both periods: Y_1^ℓ and Y_2^ℓ
  - Is born without assets
  - Derives utility from the discounted value of consumption

    More on this later
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Budget Constraint

Let's work out household balance sheet values for each period:

Period 1

Nominal Income: $P_1 Y_1^\ell$

Nominal consumption: $P_1 C_1$

Savings: $X_2 = P_1 Y_1^\ell - P_1 C_1$

Period 2

Cash in period: $P_2 Y_2^\ell + (1+i)X_2$

Consumption:

$$\begin{aligned} P_2 C_2 &= P_2 Y_2^\ell + (1+i)X_2 \\ &= P_2 Y_2^\ell + (1+i)(P_1 Y_1^\ell - P_1 C_1) \end{aligned}$$

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Moving closer to a budget constraint

- Now we have (in real period 2 value):

$$C_2 = Y_2^\ell + (1+r)(Y_1^\ell - C_1)$$

- Let's collect consumption on the left and income on the right:

$$(1+r)C_1 + C_2 = (1+r)Y_1^\ell + Y_2^\ell$$

- And if we divide by $(1+r)$ to get an expression denominated in **today's real value**

$$\underbrace{C_1 + \frac{1}{1+r}C_2}_{\text{Present value of consumption}} = \underbrace{Y_1^\ell + \frac{1}{1+r}Y_2^\ell}_{\text{Present value of income}}$$

  This is the **life-time budget constraint**!

- Your lifetime income determines consumption possibilities

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The budget line

- The C_1 intercept:

$$Y_1^\ell + \frac{Y_2^\ell}{1+r}$$

what if we use all our resources to consume C_1, and thus set C_2 to zero

- The C_2 intercept:

$$(1+r)Y_1^\ell + Y_2^\ell$$

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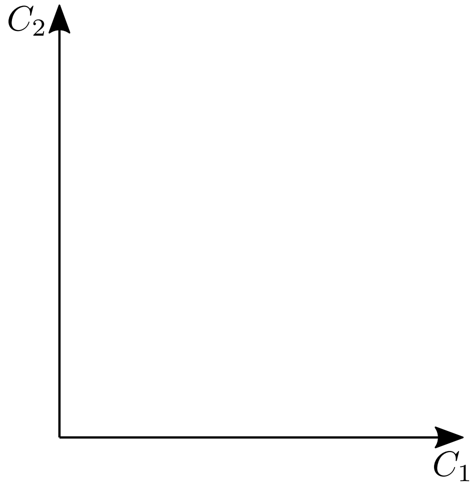
- The slope of the budget line:

$$-(1+r)$$

if I increase current consumption by one unit I lose out on $-(1+r)$ units in the next period



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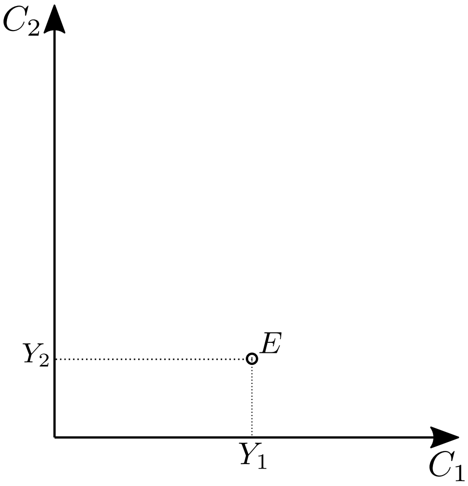

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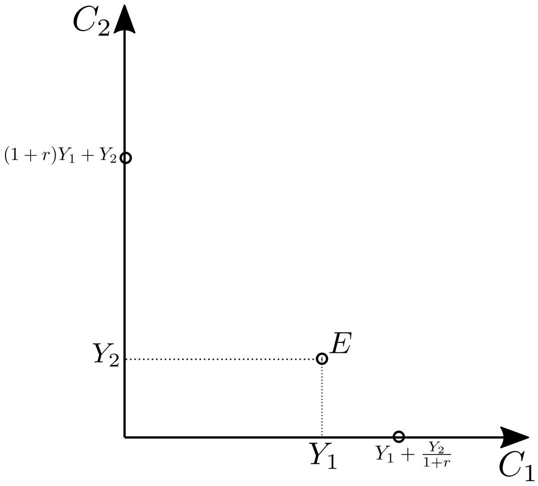

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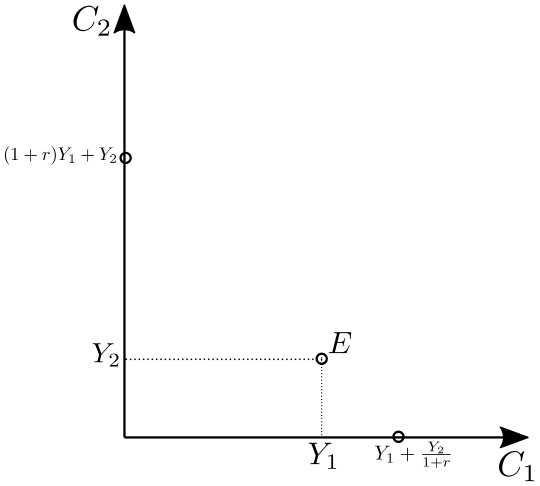

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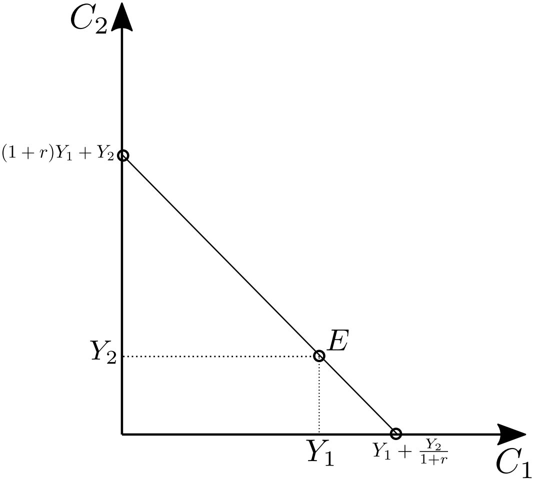

The two-period problem we are going to solve

How much does a consumer who lives for two periods consume in each period?

We need to figure out:

✓ what the consumer can afford

2. what the consumer wants to do, given what she can afford

Utility function

- What the consumer wants to do depends on her utility function.

$$U(C_1, C_2) = u(C_1) + \frac{1}{1+\rho} u(C_2)$$

- Life-time utility is the sum of current utility plus discounted utility from future consumption
- $u(\cdot)$ we often call per-period utility function, or instantaneous utility function
- Assumptions we make about $u(\cdot)$:
  - More consumption gives more utility: $\frac{\partial u}{\partial C} = u'(C) > 0$
  - Marginal utility is decreasing: $\frac{\partial^2 u}{\partial C^2} = u''(C) < 0$
- $\Rightarrow$ The consumer wants to smooth consumption over time

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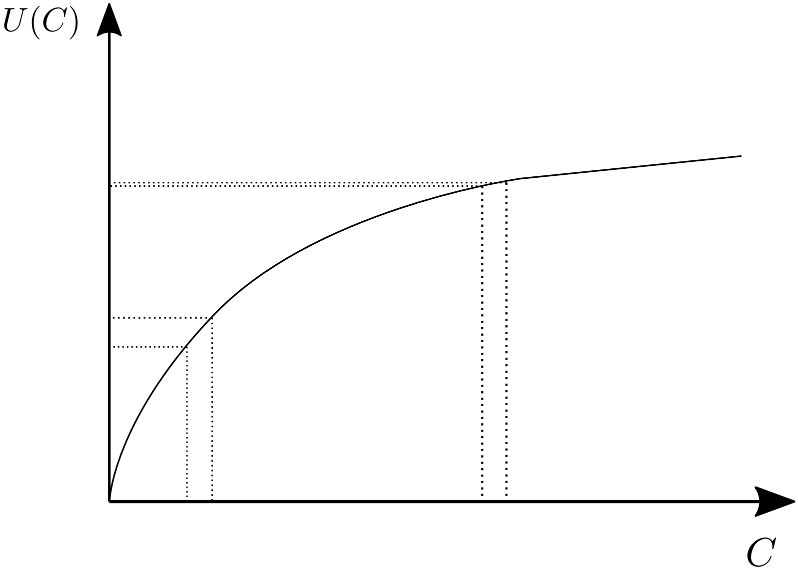
The per-period utility function



Utility example

- Assume that $u(c) = \log(c)$ and $\rho = 0$

$$U(C_1, C_2) = \log C_1 + \log C_2$$

- Does the utility function satisfy the assumptions we made? Let us check!

$$u'(c) = \frac{\partial u}{\partial c} = \frac{1}{c} > 0 \qquad OK!$$

$$u''(c) = \frac{\partial^2 u}{\partial c^2} = -\frac{1}{c^2} < 0 \qquad OK!$$

- We have 200 units in total to consume. Which choice is better?

Alternative 1: $C_1 = 100, C_2 = 100$

$U = \log 100 + \log 100 = 4.61 + 4.61 = 9.22$

Alternative 2: $C_1 = 150, C_2 = 50$

$U = \log 150 + \log 50 = 5.01 + 3.91 = 8.92$

- $\implies$ It's better to smooth consumption

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- Does the utility function satisfy the assumptions we made? Let us check!

$$u'(c) = \frac{\partial u}{\partial c} = \frac{1}{c} > 0 \qquad OK!$$

$$u''(c) = \frac{\partial^2 u}{\partial c^2} = -\frac{1}{c^2} < 0 \qquad OK!$$

- We have 200 units in total to consume. Which choice is better?

Alternative 1: $C_1 = 100, C_2 = 100$

$U = \log 100 + \log 100 = 4.61 + 4.61 = 9.22$

Alternative 2: $C_1 = 150, C_2 = 50$

$U = \log 150 + \log 50 = 5.01 + 3.91 = 8.92$

- $\implies$ It's better to smooth consumption

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The subjective discount rate ρ

$$U(C_1, C_2) = \log C_1 + \frac{1}{1+\rho} \log C_2$$

$$U(C_1, C_2) = u(C_1) + \frac{1}{1+\rho} u(C_2)$$

What is ρ?

- The *subjective discount rate* ρ tells us how much the consumer values the future
- It is *not* an observable market rate but a parameter which describes preferences
- Large ρ means that the consumer is impatient and cares little about the future
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Solving the consumer problem

- Now we know enough to write down the consumer's problem:

$$\max_{C_1, C_2} \; u(C_1) + \frac{1}{1+\rho} u(C_2)$$
$$\text{s.t. } C_1 + \frac{1}{1+r} C_2 = Y_1^\ell + \frac{1}{1+r} Y_2^\ell$$

- Let's write down the Lagrangian:

$$\mathcal{L} = u(C_1) + \frac{1}{1+\rho} u(C_2) - \lambda \left[C_1 + \frac{1}{1+r} C_2 - Y_1^\ell - \frac{1}{1+r} Y_2^\ell \right]$$

- Now we can take first order conditions:

$$[C_1] \qquad u'(C_1) = \lambda$$
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- This is called the **Euler Equation**. Consumers equalize marginal utility in all periods

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Visual intuition

- We have our Euler Equation:

$$\frac{u'(C_1)}{u'(C_2)} = \frac{1+r}{1+\rho}$$

- Slope of the indifference curve:

$$MRS = -\frac{MU_{C_1}}{MU_{C_2}} = -\left[\frac{u'(C_1)}{\frac{1}{1+\rho}u'(C_2)}\right]$$
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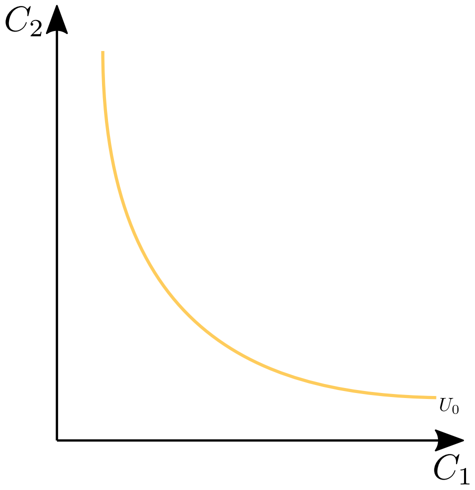

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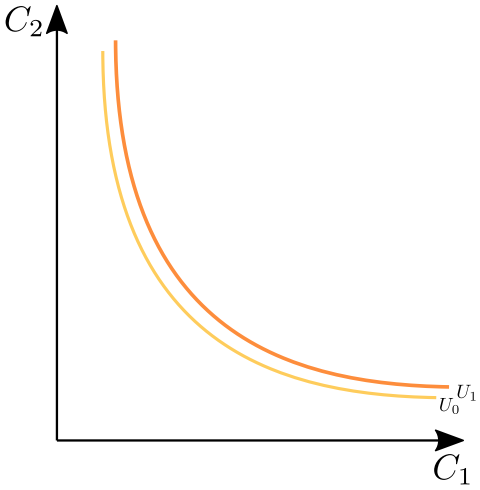

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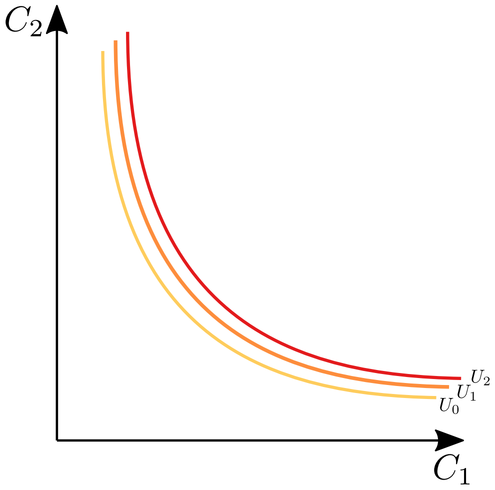

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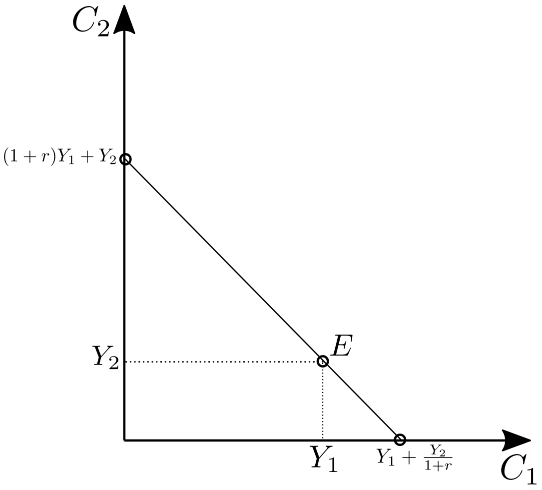

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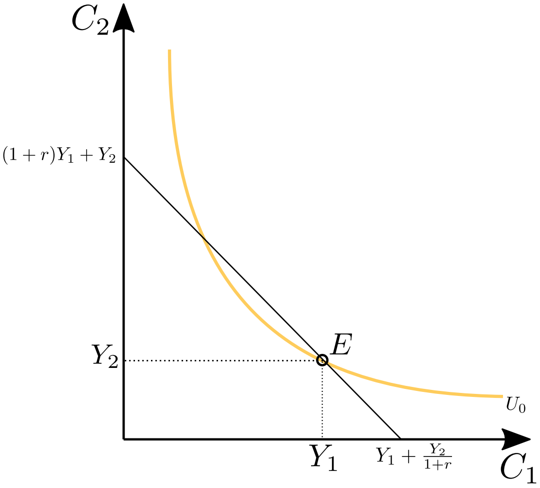

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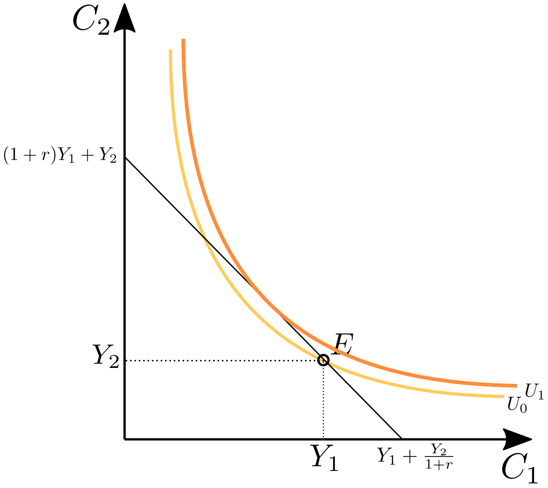

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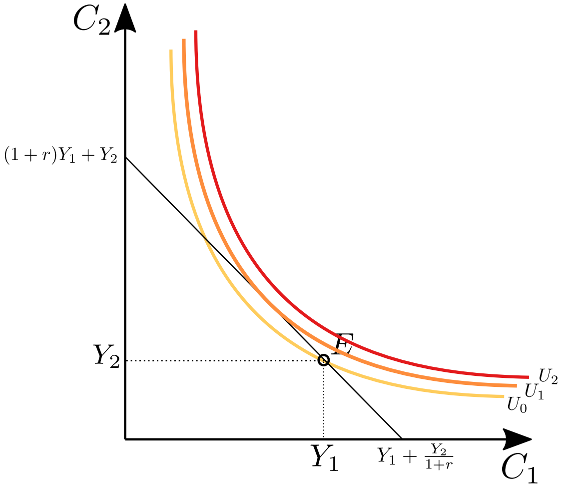

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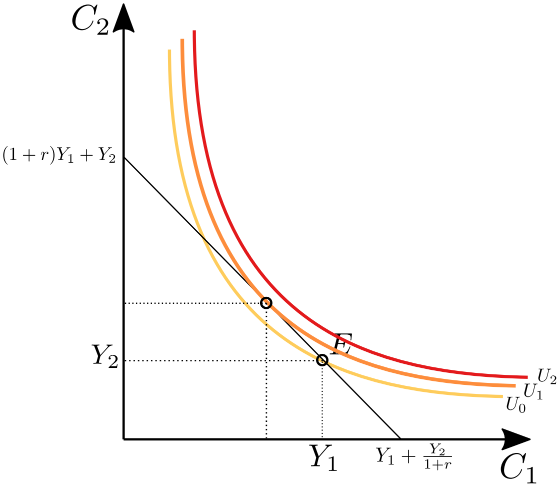

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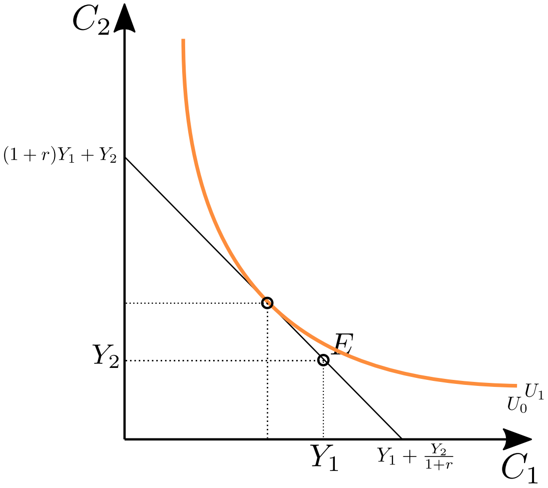
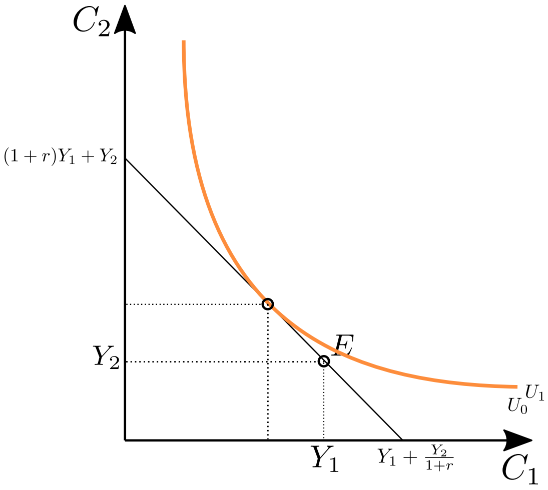


What role does the point *E* play?

E is the **endowment point**.

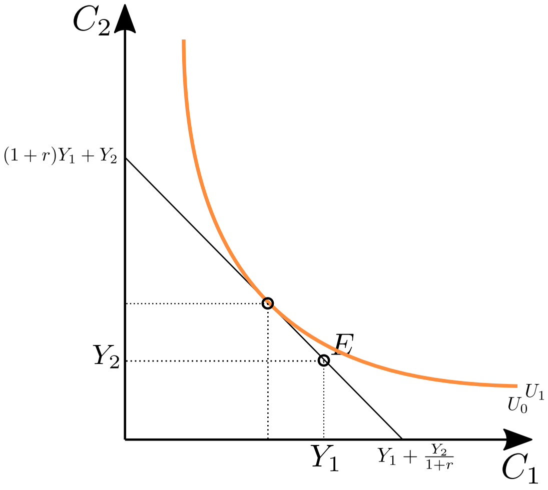
This is the amount the consumer gets (in each period), and that she can always choose to consume, without "trading" (without saving or borrowing), so it is not affected by the interest rate *r*.

- **Saver**: if the chosen consumption point is to the left of *E*
- **Borrower**: if the chosen consumption point is to the right of *E*



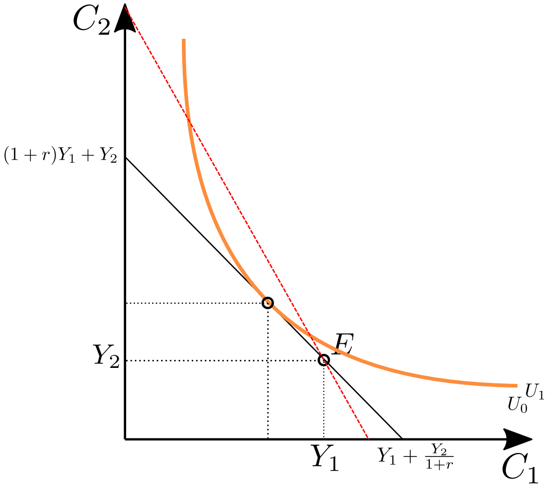
What role does r play?

- r affects the slope of the budget constraint (slope: $-(1+r)$)
- An increase in r makes the budget line steeper (more negative)
- A decrease in r makes the budget line flatter (less negative)
- However: the consumer can always choose to consume at E. This is why the budget constraint "rotates" around this point.
- The endowment point is always a possible consumption bundle.



What happens if the interest rate *r* increases?

- There has been a change in the relative price of consumption today vs. tomorrow.
- C_1 has become relatively more expensive compared to C_2.
- Make sense to swap some C_1 for some C_2, since C_2 is cheaper
- This is the **substitution effect**. Effect is unambiguous
- However, the **income effect** is ambiguous: it depends on whether the consumer is a net borrower or net saver



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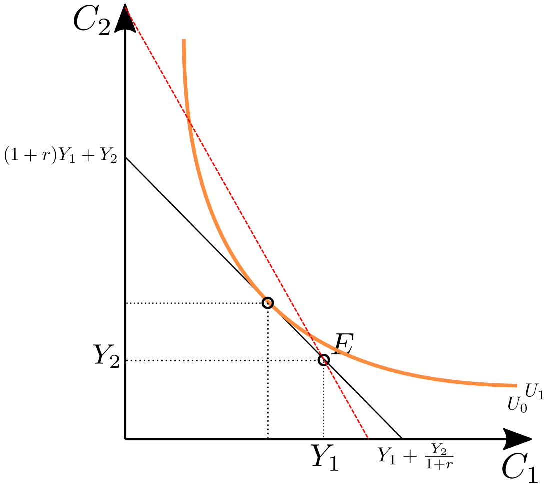

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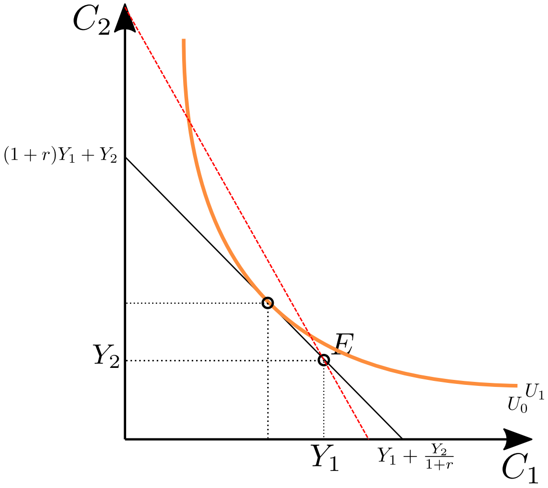

What happens if the interest rate *r* increases?

- What is the effect of an increase in the real interest rate when the consumer is a ***saver***?
- **Substitution effect:** C_1 has become more expensive relative to C_2. Consumer wants less C_1 and more C_2.
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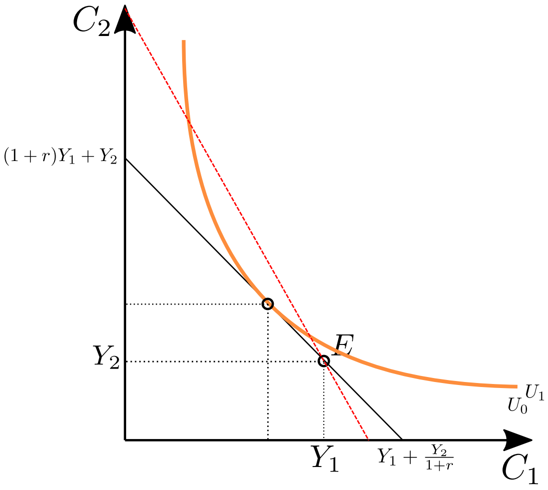
Effect	C_1	C_2
Substitution effect	↓	↑
Income effect	↑	↑
Net effect	?	↑



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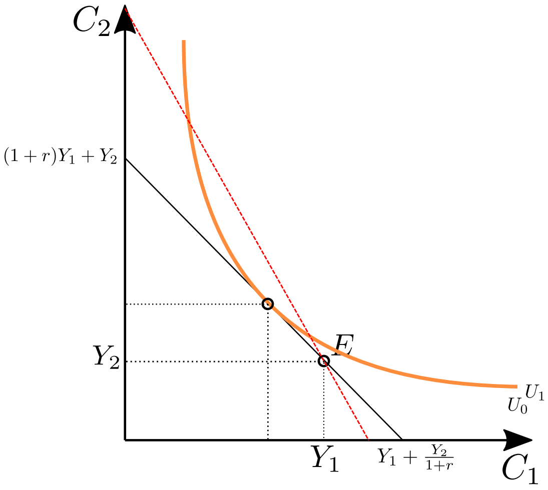

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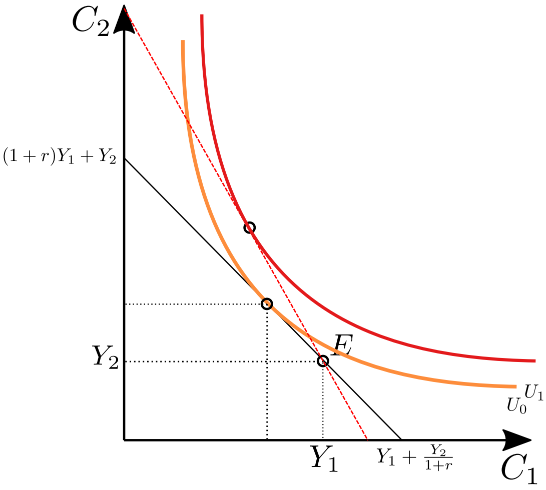

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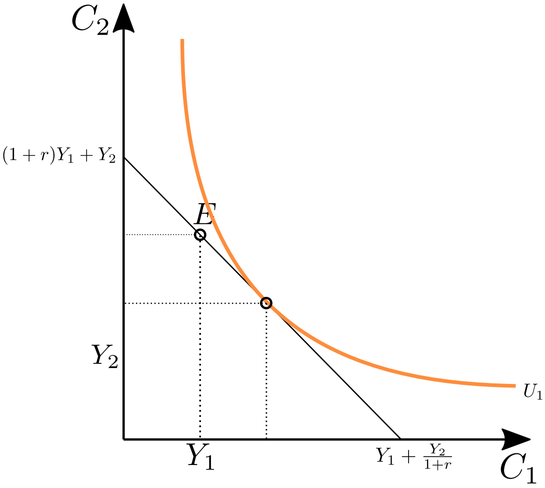

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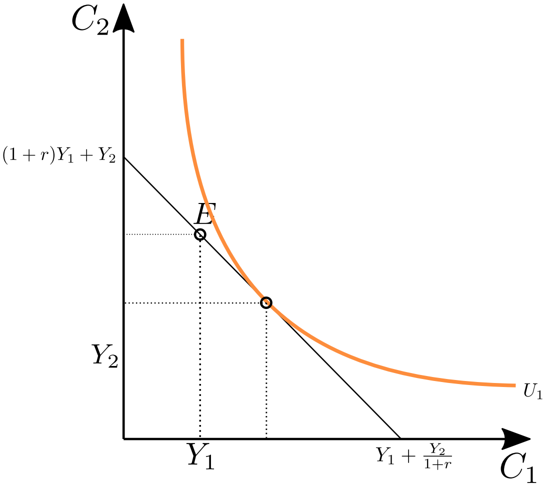

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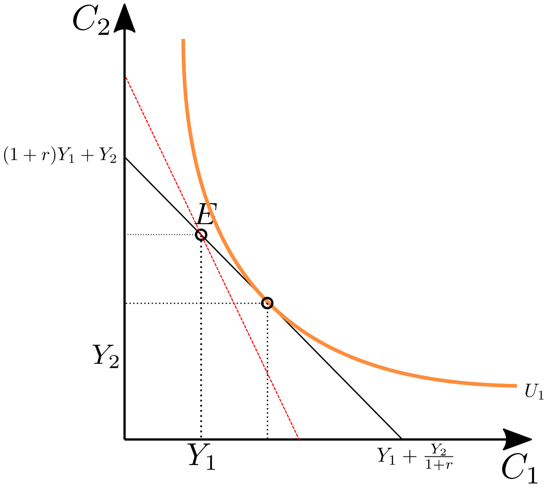

Consumption today or tomorrow? The role of ρ

What is the relationship between C_1 and C_2? Use the Euler Equation!

$$u'(C_1) = \frac{1+r}{1+\rho} u'(C_2)$$

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  This means that $u'(C_1) < u'(C_2)$, which is only true if $C_1 > C_2$

- $\rho < r$: We get a great interest rate if we save, and we are not very impatient.

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Summary

We have now worked through a 2-period model.

In this small model, consumption in the first period depends on the following objects which can be thought of as exogenous (to the individual consumer):

- Y_1 Income in period 1: The more income in period one, the more we consume (+)
- Y_2 Income in period 2: The more income in period two, the more we consume (+)
- r The interest rate: the effect of an interest rate increase depends on if we are savers (then ambiguous effect) or borrowers (then negative effect) (+/-)

It also (of course) depends on ρ and the functional form of $u(\cdot)$, but those we take as given.

Table of Contents

Infinite horizon

- Having considered a consumer who lives for only two periods, let us consider a more "realistic" case: a consumer who lives forever.

  Technically there's an intermediate case (overlapping generations with finite horizons). You'll learn about this in future courses, but it adds a lot of complication without changing the key story too much.

- What is the lifetime utility for this consumer?

$$\begin{aligned} U_t &= u(C_t) + \frac{u(C_{t+1})}{1+\rho} + \frac{u(C_{t+2})}{(1+\rho)^2} + \frac{u(C_{t+3})}{(1+\rho)^3} + \dots \\ &= \sum_{s=0}^{\infty} \frac{u(C_{t+s})}{(1+\rho)^s} \end{aligned}$$

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- In our 2-period model the lifetime budget constraint looked like:

$$\underbrace{C_1 + \frac{1}{1+r}C_2}_{\textit{present value of consumption}} = \underbrace{Y_1^\ell + \frac{1}{1+r}Y_2^\ell}_{\textit{present value of income}}$$

- Now, with infinite number of periods, it looks like:

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- We can write this out with summation notation as well

$$\sum_{t=1}^{\infty}\left[\prod_{s=1}^{t}\left(\frac{1}{1+r_t}\right)\right]C_t = \sum_{t=1}^{\infty}\left[\prod_{s=1}^{t}\left(\frac{1}{1+r_t}\right)\right]Y_t^\ell$$

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- It turns out to be much easier (and equivalent) to work with the period-by-period budget constraint
- Think about a consumer in time t:
  - They have some amount of money in the bank (X_t) (that they deposited last period) on which they even got some interest rate (i_{t-1})
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  - They consume something ($P_t C_t$)
  - Whatever is left after these events, they puts back into their bank account
- Let X_t be the "nominal stock of assets" that a consumer has at time t.
- The stock of assets changes according to the following equation:

$$X_{t+1} = P_t Y_t^{\ell} - P_t C_t + (1 + i_{t-1}) X_t$$

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The infinite horizon problem

- We're now in a position to write down the formal maximization problem that the consumer solves:

$$\max_{C_t, A_t} \quad \sum_{t=1}^{\infty} \left(\frac{1}{1+\rho} \right)^{t-1} u(C_t) \tag{1}$$

$$\text{s.t.} \quad A_{t+1} = Y_t^{\ell} - C_t + (1+r_t)A_t \qquad \text{for all } t$$

- Challenge: this is an infinite dimensional problem. We need to choose C_t and A_t for all $t = 1, 2, 3, \ldots$ You're going to learn how to properly handle problems like this next year.

- In practice, we just have to write out the Lagrangian (appropriately defined) and take first order conditions like before. If you want to learn more about this, come see me in office hours

- When we do that, we can show that optimality requires an **Euler equation** that looks *exactly* like the two period case:

$$u'(C_t) = \frac{1+r_t}{1+\rho} u'(C_{t+1}) \qquad \text{for all } t = 1, 2, 3, \ldots \tag{2}$$

Infinite model

- To continue to make some progress, we will make three assumptions:
  1. The income each period is constant ($Y_t^\ell = Y^\ell$ for all t)
  2. The interest rate is constant ($r_t = r$ for all t)
  3. $1 + r = 1 + \rho$
- The last assumption in practice means that the consumer will want to have the same consumption every period!
- Why? Look at the Euler Equation

$$\frac{u'(C_t)}{u'(C_{t+1})} = \frac{1+r}{1+\rho} = 1 \implies u'(C_t) = u'(C_{t+1}) \implies C_t = C_{t+1}$$

- **Claim:** The only solution to the consumer's problem requires that assets are constant:

$$A_t = A_{t+1} = \cdots = A_{t+i} = A$$

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Constant (sustainable) level of consumption

- To verify the claim that we need $A_t = A$ for all t, start from the per-period budget constraint:

$$A_{t+1} = Y^\ell - C + (1+r)A_t$$

- rewrite it as an expression of (constant) consumption:

$$\begin{aligned} C &= Y^\ell + (1+r)A_t - A_{t+1} \\ &= Y^\ell + rA_t - (A_{t+1} - A_t) \end{aligned}$$

- To hold consumption constant, there are two options:
  1. $A_t = A_{t+1} = \cdots = A_{t+i} = A$
  2. $rA_t = A_{t+1} - A_t$
- If 2 holds, then assets/debts will be growing without bound over time. It can't make sense to have assets go off to infinity (why would you keep constant consumption with infinite assets?), so this can't be the case.

  Note: This is called a **transversality condition**, but you'll learn more about this next year
- Therefore, we must have constant assets

Constant (sustainable) level of consumption

$$C = Y^{\ell} + rA$$

In every period, we consume:

- Our income (which is constant)
- The interest payment for our assets (which are constant)

Our consumption is constant!

Note that our assets can be positive or negative...

Other solutions (depending on ρ)

Note that this sustainable level of consumption is the optimal choice for a consumer who has a subjective discount rate which is equal to the market interest rate ($\rho = r$).

A more impatient consumer ($\rho > r$) will consume more than this, assets will decrease over time, consumption will decrease, and they will become poor.

Conversely, a patient consumer who has a value of ρ which is lower than the market interest rate ($\rho < r$) will consume less than her sustainable level and will accumulate assets.

We keep the assumption that $\rho = r$ for now!

Summing up the inifinite horizon case

We can conceptually write our consumption function in the infinite-horizon model as:

$$C = C(Y^{\ell}, A, r)$$

Consumption is a function of

- Y^{ℓ} The constant labour income: The higher the income, the more we consume (+)
- A The asset level: The more assets, the more we consume (+)
- r The interest rate: the effect of an interest rate increase depends on if we are savers (then positive effect) or borrowers (then negative effect) (+/-)

Table of Contents

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- A consumption function expresses consumption expenditures as a function of objects which can be thought of as exogenous (to individual consumers).

- The purpose of a consumption function is to:
  1. summarise concisely which factors cause current C to vary
  2. provide a simple tool to model consumption in general eq.

- We need something more realistic than our two-period model, but the mathematical complications make the infinite horizon model too hard to work with

  We had to make some very unpalatable assumptions to make progress, although next year you'll see how to avoid that

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"Our" consumption function

The consumption function that we will use combines insights from our two-period case and the infinite-horizon case:

$$C = C(Y, Y^{\ell,e}, r, A)$$

Consumption depends on:

- Y Income today (+)
- $Y^{\ell,e}$ Expected future permanent labour income (+)
- r The interest rate (+/-)
- A Assets (+)

But how important are these different factors? Useful to analyse a specific consumption function.

A specific consumption function

We will now derive a very specific consumption function.

Also derived in the appendix of Chapter 4

We will first make some simplifying assumptions

- From period $t+1$ onwards, labor income is constant: $Y^{\ell,e}$
- From period $t+1$ onwards, the real interest rate is constant: $\bar{r}$
- This constant real interest rate is equal to the subjective discount rate: $(1+\rho) = (1+\bar{r})$

This means that from period $t+1$ onwards, we will be at our constant, sustainable, consumption level:

$$C_{t+1} = Y^{\ell,e}_{t+1} + \bar{r}A_{t+1} \tag{3}$$

Budget constraint

- But what about this period, period t?
- From the infinite-horizon model of consumption we know that the per-period budget constraint for the consumer is

$$A_{t+1} = Y_t^{\ell} - C_t + (1 + r_t)A_t \tag{4}$$

- Let us define a new variable now which combines both labor income and asset income into one single variable that we (due to lack of imagination) call *income*

$$Y_t = Y_t^{\ell} + r_t A_t$$

- Thus, we can write the per-period budget constraint as:

$$A_{t+1} = Y_t + A_t - C_t$$

Budget constraint

This means that assets in $t+1$ will be

$$A_{t+1} = Y_t + A_t - C_t$$

So this is how much we will save for next period, time $t+1$.

What will happen then? If labor income is expected to be constant in $t+1$ it means that consumption from next period onwards is expected to be on the sustainable constant level

$$C_{t+1} = Y_{t+1}^{\ell,e} + \bar{r}A_{t+1}$$

Using this and substituting into the first equation:

$$C_{t+1} = Y_{t+1}^{\ell,e} + \bar{r}(Y_t + A_t - C_t)$$

Utility

How do we make sure that the consumer is maximizing her utility? Use the Euler equation!
Remember:

$$u'(C_t) = \frac{1 + r_{t+1}}{1 + \rho} u'(C_{t+1})$$

In this case, we assume log utility, and then it is very simple:

$$u(C_t) = \log C_t \quad \Rightarrow \quad u'(C_t) = \frac{1}{C_t}$$

Our Euler equation becomes:

$$\frac{1}{C_t} = \frac{1 + r_{t+1}}{1 + \rho} \frac{1}{C_{t+1}}$$

which we can rewrite as:

$$C_{t+1} = \frac{1 + r_{t+1}}{1 + \rho} C_t$$

Finding the consumption function

The budget constraint gives us:

$$C_{t+1} = Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t - C_t)$$

Utility maximization gives us:

$$C_{t+1} = \frac{1 + r_{t+1}}{1 + \rho} C_t$$

Let's combine them and solve for C_t!

$$\frac{1 + r_{t+1}}{1 + \rho} C_t = Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t - C_t)$$

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Remove C_t from the brackets on the right hand side ...

$$\frac{1+r_{t+1}}{1+\rho}C_t = Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t) - \bar{r}C_t$$

... and then bring it to the left hand side ...

$$\frac{1+r_{t+1}}{1+\rho}C_t + \bar{r}C_t = Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t)$$

... and then factor it out:

$$C_t\left(\frac{1+r_{t+1}}{1+\rho} + \bar{r}\right) = Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t)$$

$$C_t = \frac{Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t)}{\left(\frac{1+r_{t+1}}{1+\rho} + \bar{r}\right)}$$

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So, what has this bought us?

We have created a specific version of a **consumption function**:

$$C = C(Y, Y^{\ell,e}, r, A)$$

…that helps us understand how consumption is determined, also quantitatively!

Finding the consumption function

$$C_t = \frac{Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t)}{\left(\frac{1+r_{t+1}}{1+\rho} + \bar{r}\right)}$$

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Comparative statics of the consumption function

$$C_t = \frac{Y^{\ell,e}_{t+1} + \overline{r}(Y_t + A_t)}{\left(\frac{1+r_{t+1}}{1+\rho} + \overline{r}\right)} \approx \frac{Y^{\ell,e}_{t+1} + \overline{r}(Y_t + A_t)}{(1+\overline{r})}$$

How does the consumption depend on:

1. Y_t, **current income:**

$$\frac{\partial C_t}{\partial Y_t} \approx \frac{\overline{r}}{1+\overline{r}} \approx \overline{r}$$

A unit increase in current income raises consumption by a fraction $\overline{r}$ – because consumers prefer to spread the consumption benefit over time.

Comparative statics of the consumption function

$$C_t = \frac{Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t)}{\left(\frac{1+r_{t+1}}{1+\rho} + \bar{r}\right)} \approx \frac{Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t)}{(1+\bar{r})}$$

How does the consumption depend on:

2. A_t, **assets (current wealth):**

$$\frac{\partial C_t}{\partial A_t} \approx \frac{\bar{r}}{1+\bar{r}} \approx \bar{r}$$

A unit increase in current assets raises consumption by a fraction $\bar{r}$ – because consumers prefer to spread the consumption benefit over time.

Comparative statics of the consumption function

$$C_t = \frac{Y_{t+1}^{\ell,e} + \bar{r}(Y_t + A_t)}{\left(\frac{1+r_{t+1}}{1+\rho} + \bar{r}\right)} \approx \frac{Y_{t+1}^{\ell,e} + \bar{r}(Y_t + A_t)}{(1+\bar{r})}$$

How does the consumption depend on:

3. $Y_{t+1}^{\ell,e}$, **expected permanent labour income:**

$$\frac{\partial C_t}{\partial Y_{t+1}^{\ell,e}} \approx \frac{1}{1+\bar{r}} \approx 1$$

The effect of an increase in permanent income is close to one.

Comparative statics of the consumption function

$$C_t = \frac{Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t)}{\left(\frac{1+r_{t+1}}{1+\rho} + \bar{r}\right)} \approx \frac{Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t)}{(1+\bar{r})}$$

How does the consumption depend on:

4. r_{t+1}, **the real interest rate we expect next period:**

$$\frac{\partial C_t}{\partial r_{t+1}} = -\left(\frac{Y^{\ell,e}_{t+1} + \bar{r}(Y_t + A_t)}{\left(\frac{1+r_t}{1+\rho} + \bar{r}\right)^2 (1+\rho)}\right) < 0$$

Raising the interest rate today tends to make you more likely to save for the future

Summary

- The consumption function expresses consumption expenditures as a function of objects which can be thought of as exogenous (to the individual consumer)
- The purpose of using a consumption function is to:
  1. Summarise concisely which factors cause current C to vary
  2. Provide a simple tool to model consumption in general eq.
- We will work with a consumption function of the form

$$C = C(Y, Y^{\ell,e}, r, A)$$

- Much of the intuition to understand the consumption function and what it depends on is gained from our 2-period example and our simplified infinite horizon example