Interest Rates and Investment

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Slides adapted from Jonna Olsson

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Where we've been:

- Last week, we developed the theory of how production works in the economy
- We thought through how output depends on the basic factors of production
- We analyzed the determinants of the "natural rate" of output

Where we're going:

- One of the main ways that households and firms interact is through the **financial market**
- ▶ When households save, the only place they have to invest those savings is through the firms
- This week, we're going to be thinking through the demand side of this market (firm investment)
- Next week, we're going to discuss the supply side (the consumer savings decision)
- However, we first need to discuss the key price that functions to "clear" the market:

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Aggregate demand

Y = C + I

- ▶ In our simplified model (no government, closed economy), aggregate demand is the sum of
 - Private consumption (made by households)
 - Private investment (made by firms)
- Both these decisions are about allocating resources over time, they are *intertemporal* decisions!
 - A consumer who puts off consumption today, and saves instead, can consume more tomorrow
 - A firm that invests today produces more goods and services in the future
- For intertemporal decisions the price of "moving resources between time periods" is key.
 - This price is *the interest rate*.
 - The interest rate in monetary terms is called the nominal interest rate, but it is the real interest rate the interest rate measured in terms of goods that matters for consumption and investment decisions.

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- Consider borrowing £1 today and paying it back next year with interest
- Let i_t denote the **nominal interest rate** from period t to t + 1
- If you borrow 1 today, you must pay $1 + i_t$ next year
- > Therefore, one plus the interest rate is the price of money today, in terms of money next year
- How much do I have to save today to have one unit of money in my bank account next year?

▶ If I put that amount in the bank account now, I will next year have:

$$\frac{1}{1+i_t}(1+i_t) =$$

- The price of money next year in terms of money today, $1/(1+i_t)$, is called the *discount factor*
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- We will simplify our analysis and **assume** there is only one, which we call i_t

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- The price of the unit is P_t today and P_{t+1} next year
- What is the cost, in terms of money today, of consuming one unit today or the next period?

• We need $P_{t+1}/(1+i_t)$ in our bank account today.

Then we will have $\frac{P_{t+1}}{1+i_t}(1+i_t) = P_{t+1}$ tomorrow, right?

> This means the price of consumption today in terms of consumption in the next period is

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- ▶ If the nominal interest rate is 5% and inflation is zero then the relative price is 1.05
- If the nominal interest rate is 5% and inflation next period is 3% then the relative price is 1.019 (which is approximately 1.02)

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Real interest rate

- If the nominal interest rate is higher than the inflation, the relative price of consumption is greater than 1
- This means that you get more than one unit of consumption next period if you abstain one unit today
- We measure this with **the real interest rate**:

$$1 + r_{t+1} = \frac{1 + i_t}{1 + \pi_{t+1}}$$

This is the **real return** you get after taking inflation into account.

- A note on timing:
 - i_t The interest rate that is agreed on today, time t, when you put your money in the bank
 - π_{t+1} Inflation that was realized in period t+1
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Timing indices tell you the period when the variable can actually be measured (uncertainty is resolved)

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You have £10. One Snickers is £0.5, so you have the value of 20 Snickers today. But are you that hungry, really? There will be a Snickers opportunity next year as well.

Scenario 1:

- Put the money in a bank account, get an interest rate of 10%
- ▶ Next year Snickers cost £0.55
- ▶ You'll buy $10 \cdot (1 + i_t)/0.55 = 20.0$ Snickers next year

- Put the money in a bank account, get an interest rate of 7%
- Next year Snickers cost £0.51
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Scenario 1:

- Nominal interest rate $i_t = 10\%$
- ▶ Inflation $\pi_{t+1} = 10\%$
- ▶ Real interest rate: $r_{t+1} = 0\%$

- ▶ Nominal interest rate $i_t = 7\%$
- ▶ Inflation $\pi_{t+1} = 2\%$
- ▶ Real interest rate $r_{t+1} = 5\%$

Real interest rate: Rule of Thumb

For small values of i_t and π_t , the following approximation is useful:

 $r_{t+1}\approx i_t-\pi_{t+1}$

Why can we do this? We have $1 + r_{t+1} = \frac{1+i_t}{1+\pi_{t+1}}$. Thus:

$$(1 + r_{t+1})(1 + \pi_{t+1}) = 1 + i_t$$

$$1 + r_{t+1} + \pi_{t+1} + r_{t+1}\pi_{t+1} = 1 + i_t$$

$$r_{t+1} + \pi_{t+1} + r_{t+1}\pi_{t+1} = i_t$$

But if both r_{t+1} and π_{t+1} are small, their product is very very small and can be ignored. Example: if $r_{t+1} = 0.03$ and $\pi_{t+1} = 0.02$, then $r_{t+1}\pi_{t+1} = 0.0006$.

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i_t and π_{t+1}

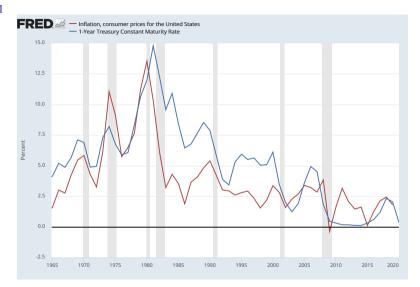


Figure: Source: https://fred.stlouisfed.org/

Summary nominal vs. real interest rate

The nominal interest rate, *i_t*, is the currency unit denominated return on an asset over a given period of time

• The real interest rate strips out the effect of inflation: $r_{t+1} \approx i_t - \pi_{t+1}$

- > The real interest rate is the purchasing power equivalent interest rate
- The real interest rate affects the trade-off between current consumption and future consumption, or investing and not investing
- If nominal prices adjust slowly, then (policy induced) nominal interest rate movements alter the real interest rate

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What is investment?

- Investment (in Macro) is not just a reallocation of assets among different individuals
- Investment, as macroeconomists use the term, creates a new physical asset
- > That asset is called capital and it is used in the production of goods and services
- **Example 1**: People often think about financial market when they think of investment. Is buying stock in a company considered investment?
 - No, that is not considered investment, since nothing new has been created in this transaction
 - Money (ownership) has just been moved around within the economy
- **Example 2**: Let's imagine that my two options for buying a house are
 - 1. A new modern house that I build in the countryside
 - 2. Buying a previously built nice house from someone

The first option would add a house to the economy. The second option merely reallocates an existing house from someone else to me. Therefore, only the first option would be counted as an investment.

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Categories of investment

We will look at three different types of investment:

Business fixed investment

Purchase of new plants and equipment by firms

Residential fixed investment

The purchase of new housing by households and landlords

Inventory investment

Increase in the firms' inventories of goods

Investment creates capital:

- As we saw in Chapter 2, capital is an important production factor and the capital stock affects the long-run level of income
- At a given point in time, the capital stock is fixed, determined by actions and events that happened in the past (*stock variable*)

The capital stock changes for two reasons:

- Investment adds new buildings and machines to the capital stock
- Depreciation reduces the capital stock as machines break down or become obsolete, buildings have to be torn down or repaired, and so on

- Let K_t denote the capital stock at the beginning of period t
- We assume that this stock can be used for production in period t but that any capital that is added during period t can only be used for production in period t + 1
- $\blacktriangleright\,$ We also assume that the capital stock depreciates at a constant rate, $\delta\,$
- > The following equation (law of motion) shows how the capital stock changes over time:

$$K_{t+1} = K_t + \underbrace{I_t - \delta K_t}_{t+1}$$

net investment

- Let K_{t+1}^d denote the desired capital stock in period t + 1
 - Assume that investment in period t is made to reach the desired level of capital in t + 1
 - ▶ Then we can decompose investments into two pieces:

$$I_{t} = \underbrace{K_{t+1}^{d} - K_{t}}_{lographic track} + \underbrace{\delta K_{t}}_{replacing the depresented track}$$

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$$I_{t} = \underbrace{K_{t+1}^{d} - K_{t}}_{\text{Increasing the capital stock}} + \underbrace{\delta K_{t}}_{\text{replacing the depreciation}}$$

We want to figure out how the firm will choose K^d . We need to make some assumptions:

- ▶ The same basket of goods is used for consumption and investment
- Capital can be bought (and sold) at the price P_t
- The company finances its investments by borrowing from households at interest rate i_t
- Capital bought in period t can be used for production in period t + 1

Now we can ask: What is the effect on profit of an increase in K_{t+1} by one unit?

Period t: The company borrows and invests. No effect on profit

Period t + 1: Profits are affected in three ways

$$\underbrace{MPK_{t+1} \cdot MR_{t+1}}_{\text{Increase in revenue}} - \underbrace{(1+i_t)P_t}_{\text{Repayment of loan}} + \underbrace{(1-\delta)P_{t+1}}_{\text{Remaining market value}}$$

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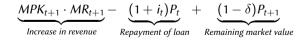
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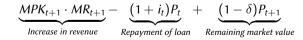
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Profit maximization requires:

$$MPK_{t+1} \cdot MR_{t+1} - (1+i_t)P_t + (1-\delta)P_{t+1} = 0$$

Note that this equation is denominated in prices This is the profit next period, in next period's money.

Divide by P_{t+1} everywhere (to get an expression in next period's quantities, i.e., in real terms):

$$MPK_{t+1} \cdot \frac{MR_{t+1}}{P_{t+1}} - (1+i_t)\frac{P_t}{P_{t+1}} + (1-\delta)\frac{P_{t+1}}{P_{t+1}} = 0$$

Now remember two key equations:

$$MR_{t+1} = \left(1 + \frac{1}{\eta}\right)P_{t+1} = \frac{1}{1+\mu}P_{t+1} \quad \text{(Lecture 2)}$$
$$(1+i_t)\frac{P_t}{P_{t+1}} = \frac{1+i_t}{1+\pi_{t+1}} = 1+r_{t+1} \quad \text{(Slide 7)}$$

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This gives us:

$$\frac{MPK_{t+1}}{1+\mu} - \delta = r_{t+1} \tag{1}$$

This is our key condition determining investment:

real net income from extra unit of capital = real interest rate.

Sometimes we say that the **user cost of capital** should be equal to the increase in revenue:

$$\frac{MPK_{t+1}}{1+\mu} = \underbrace{r_{t+1} + \delta}_{user\ cost\ of\ capital}$$

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Implications of the exercise we just did:

- When the marginal revenue product of capital is greater than the cost of depreciation and the cost of the loan, the firm should invest in more capital
- When the marginal revenue product of capital is less than the cost of depreciation and the cost of the loan, the firm should invest in less capital
- **Optimal investment** occurs when the marginal revenue product of capital equals the depreciation plus the real interest rate (the real cost of borrowing)

(Exactly the same logic as when we discussed pricing and profit maximization)

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What is the relevant price for intertemporal decisions?

What determines the demand for investment?

What is the demand for capital in the long and short run?

Why are investments so volatile?

What about inventories and housing?

The demand for capital

• We have figured out the key equation for the optimal capital stock:

$$\frac{MPK_{t+1}}{1+\mu} = r_{t+1} + \delta$$

- Now we can figure out how much capital a firm wants to hold! We will do it in two scenarios:
 - The long run
 - The short run
- Let's start with the long run!
 - ▶ By long run, we mean that we assume employment is at its natural level
 - ▶ To make things easy, we will further assume Cobb-Douglas Production

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We will need to combine three pieces of insights:

1. The equation for the optimal capital stock:

$$\frac{MPK_{t+1}}{1+\mu} = r_{t+1} + \delta$$

Note: We're going to drop time subscripts when we're thinking about the long-run steady state. If the economy is in equilibrium, then $r_t = r_{t+1}$ and $MPK_t = MPK_{t+1}$.

2. Output is at its natural level, given by the natural level of employment, in the long run:

$$Y = F(K, EN^n); \quad N^n = (1 - u^n)L$$

3. The assumption about Cobb-Douglas technology:

$$Y = K^{\alpha} (EN)^{1-\alpha}$$

With a Cobb-Douglas production function the MPK, the marginal product of capital, is given by:

$$MPK = \frac{\partial Y}{\partial K} = \alpha K^{\alpha - 1} (EN)^{1 - \alpha}$$

Use this in the expression for the optimal capital stock:

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$$K = \left[\frac{\alpha}{(r+\delta)(1+\mu)}\right]^{\frac{1}{1-\alpha}} EN^n$$
(2)

- *E* **Technology level:** An improvement in technology increases the demand for capital because production becomes more profitable for a given labor input and real interest rate (+)
- *Nⁿ* **The natural rate of employment:** With more workers employed, it is optimal to increase the capital stock (+)
- α Elasticity of output with respect to capital: If capital is more important in the production process, we want more capital (+)
- μ **Mark-ups:** A higher mark-up reduces the demand for capital because firms use their monopoly power to raise prices and reduce their production (-)
- *r* **The real interest rate:** A higher real interest rate has a negative effect on the demand for capital because it raises the cost of financing investment (-)
- $\delta\,$ Depreciation rate: A higher depreciation rate reduces the desired capital stock because it reduces the net return on investments (-)

The demand for capital in the long run: comparative statics

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We want to find the demand for capital in the short run in our model.

What do we mean by short run?

Employment and production can be above or below the natural levels, depending on the level of demand

What specific assumption have we made in our model?

We still use Cobb-Douglas production technology

We will need to combine three pieces of insights:

1. The equation for the optimal capital stock:

$$\frac{MPK_{t+1}}{1+\mu} = r_{t+1} + \delta$$

2. Output is *not* necessarily at its natural level, but can be below or above (driven by temporary demand factors):

Expected demand next period: Y^e

3. The assumption about Cobb-Douglas technology:

$$Y = K^{\alpha}(EN)^{1-\alpha}$$

With Cobb-Douglas production the marginal product of capital is given by:

$$MPK = \alpha K^{\alpha - 1} (EN)^{1 - \alpha} = \alpha K^{\alpha - 1} (EN)^{1 - \alpha} \frac{K}{K} = \alpha \frac{Y}{K}$$
(3)

Use this in the expression for the optimal capital stock:

$$r_{t+1} + \delta = \frac{MPK_{t+1}}{1+\mu}$$
Equation (1)
$$= \frac{\alpha}{1+\mu} \frac{Y_{t+1}}{K_{t+1}}$$
Equation (3)
$$\Rightarrow \qquad K_{t+1} = \left[\frac{\alpha}{(1+\mu)(r_{t+1}+\delta)}\right] Y_{t+1}$$
Solve for K_{t+1}

Since K_{t+1} is just K^d , and $Y_{t+1} = Y^e$ (perfect foresight), we have

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The investment function in the short run

We can then write out an **investment function**:

$$I_t = K_{t+1}^d - K_t + \delta K_t$$

= $\frac{\alpha}{(r+\delta)(1+\mu)} Y^e - K_t(1-\delta)$

Investment in the short run depends on three factors: $I(r, Y^e, K)$

- r The real interest rate, which determines the required return on the investment (-)
- Y^e Expected aggregate demand, which determines how much capital is needed (+)
- ${\it K}\,$ The current capital stock, since the more capital we already have, the less need for new investments (-)

Taking α, δ, μ as constant (for sure true in the short run)

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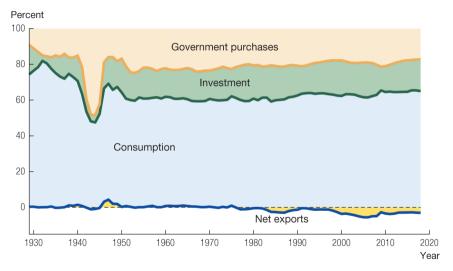
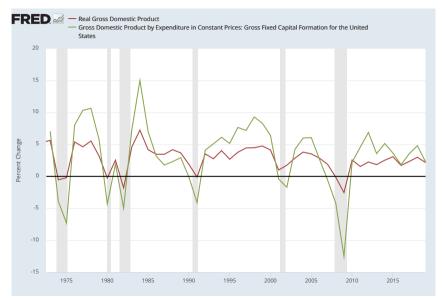


Figure: User side of GDP for the U.S. Source: Jones (2020)

Volatility of investment



The desired level of capital in the short run

We just figured out that in the short run:

$$K^d = rac{lpha}{(r+\delta)(1+\mu)}Y^e$$

How large is our desired capital stock? Assume some reasonable numbers:

$$lpha$$
 1/3
 r 5%
 δ 7%
 μ 10%
 $K^d \approx 2.5 Y^e$

- This means that the desired level of capital is to have a capital stock that is 2.5 times larger than GDP
- ▶ What happens if we expect GDP to increase by 2%?
- Our desired level of capital increases by 2% as well

Increasing the capital stock by 2% corresponds to a desire to make an investment of 5% of GDP!

- ► Our capital stock: 2.5*Y*
- $0.02 \cdot 2.5Y = 0.05Y$, i.e., 5 percent of GDP

Thus, the *expected* increase in GDP of 2% translates into an *increase in investment corresponding* to 5% of aggregate demand.

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- $0.02 \cdot 2.5Y = 0.05Y$, i.e., 5 percent of GDP

Thus, the *expected* increase in GDP of 2% translates into an *increase in investment corresponding* to 5% of aggregate demand.

- How much do we "normally" invest? Let us assume a (counterfactual) world without growth, so that we want a constant capital stock.
- Main reason for investment replace depreciation
 - How much depreciates each year?
 - Assume $\delta = 7\%$
 - $\delta K = 0.07 \cdot 2.5Y = 0.175Y$, i.e., 17.5 percent of GDP
- Thus, our "extra" investment of 5% of GDP (due to the *expected* increase in GDP of 2%) translates into an increase in investment of 29%! ((17.5 + 5)/17.5 = 1.29)
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$$\implies \frac{I_t}{Y_t} = 2.5 \left(\frac{Y_{t+1}^e - Y_t}{Y_t}\right) + 0.175$$

Desired capital stock tomorrow Assume $K_t = K_t^d$

- Investment to cover depreciation: 17.5%
- Investment to "cover growth": 2.5 times the growth rate
 - With our example: 2% growth \rightarrow 5% extra investment
 - New total: 17.5% + 5% = 22.5%, i.e. 29% more!

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The accelerator effect of investment

- Investment is highly sensitive to expected fluctuations in aggregate demand
- Fluctuations in investment contribute to realized fluctuations in demand
- Investment plays an important role in the analysis of business cycles, and has an accelerating effect

In reality, the investment accelerator effect and the volatility of investment is not as strong as we calculated it to be:

- In practice it takes time to make investment
- In practice, firms make forecasts in a smarter way: if an increase in demand is temporary, it is not as important to be "right" in the capital stock

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What determines the demand for investment?

What is the demand for capital in the long and short run?

Why are investments so volatile?

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Categories of investment

We have three different types of investment:

Business fixed investment

Purchase of new plants and equipment by firms

Residential fixed investment

The purchase of new housing by households and landlords

Inventory investment

Increase in the firms' inventories of goods

Residential fixed investment

Is building a house an investment? Yes!

- Building a house adds to the housing capital stock
- Housing capital is used in the production of goods and services: housing services

But are housing services part of GDP? Yes!

- Rent paid by tenants to landlords: obviously
- Housing services provided to people who own their own houses?
- Yes, technically they are renting it to themselves
- GDP includes "imputed" rents for owner-occupied housing

Is housing important?

- Housing investment makes up a rather small amount of overall investment
- Housing is a rather small amount of overall GDP
- BUT: it is (extremely?) important for the business cycle!

Importance of housing for the business cycle

- Demand for housing goes up when people become richer
- ...so then the housing stock has to increase
- ► The accelerator effect!
- Investment cycles in real estates can contribute to a high extent to booms and busts

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Imagine a bakery that:

- Hires workers to bake more bread
- Pays the workers their wage
- Sells the bread

How does this affect GDP?

- More production
- More income (increases the wage bill)
- More consumption (someone bought the bread)

- No increase in final consumption (no-one bought the additional bread)
- No increase in production (the spoiled bread is as if never produced)
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But what if the bakery put the bread in the freezer to sell next period?

- It is like the firm sells the bread to itself
- The frozen bread is counted as inventory investment $(I \uparrow)$

Effect on GDP?

- More production
- More income (increase in the wage bill, and profit unchanged)
- Increase in the user side, since investment increased

Next period, when the frozen bread is sold to a final consumer, it does *not* affect GDP:

- Consumption increases ($C \uparrow$)
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Different types of inventories:

- 1. Inventories of raw materials
- 2. Goods in process
- 3. Finished goods

Inventories is a type of investment in productive capital:

- ▶ In order to sell goods next period, the firm needs to hold some stock of finished goods
- In order to produce next period, the firm needs to hold some goods in process
- In order to start the production process, the firm needs to hold some raw material

Inventory investments can be:

- **Positive:** inventories increase
- Negative: inventories decrease

Again: strongly correlated to expected demand:

- If demand is expected to increase, a firm wants to increase its inventories (and vice versa)
- Accelerator effect!
- Small and sometimes even negative part of total investment (and GDP), but fluctuates quite a bit, and therefore important to analyze when thinking about business cycles

We have this week started looking at the expenditure/user side of GDP: how are the goods produced used? (starting with investments)

Y = C + I (Simplified Model)

- We figured out that the relevant price to decide if to invest or not was the real interest rate
- We derived the firm's optimality condition for the capital stock:



- > Then we figured out what the desired capital stock is in the long run
- ...and in the short run, when we do not know exactly how many people will work or what the aggregate demand will be, so we just take the expected demand as given
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