Production, Prices, and the Distribution of Income

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Firms produce output by combining inputs (factors of production)

- **Variable Factors:** inputs for production which can be scaled up and down
  - **Labor** Examples: Hours worked, human capital
  - **Capital** Examples: Machines, buildings, facilities used in production
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  - Inputs needed for production which cannot be produced easily
  - Classic example is land

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The Production Function

- The main factors of production that remain are the capital stock K and the supply of labor N

  You can think of this as number of workers employed, if you're happy to assume that each person works a fixed number of hours

- We represent the relationship between K, N, and Y with a **production function**

$$Y = F(K, N)$$

- F is meant to be a very general function. All we've said is that there is a relationship between the amount of output you can produce and the quantity of the inputs you use

  Note: This is actually a technological relationship

- Some functions might be reasonable (or "plausible") and others are not. We need to be specific about what characteristics a reasonable F will have

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Production Function: Necessary Characteristics

We will need to make three key assumptions about the "shape" of the production function:

- **Assumption 1:** Production is increasing in both K and N

$$\frac{\partial F}{\partial K} > 0 \qquad \frac{\partial F}{\partial N} > 0$$

I.e, Hiring more workers, but keeping everything else the same, should produce additional output

- **Assumption 2:** Diminishing marginal product of each input

$$\frac{\partial^2 F}{\partial^2 K} < 0 \qquad \frac{\partial^2 F}{\partial^2 N} < 0$$

Keeping the capital stock fixed, hiring your 10th worker provides less output than hiring your 1st one

- **Assumption 3:** Constant returns to scale

$$F(\lambda K, \lambda N) = \lambda F(K, N) \quad \text{For any } \lambda > 0$$

Doubling inputs doubles output

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A specific functional form

- Let's assume (for the next few slides) a specific **functional form**

$$F(K, N) = 5\sqrt{K \cdot N}$$

- If you have 30 units of capital and 30 units of labor, you produce 150 units of output:

$$F(30, 30) = 5 \cdot \sqrt{30 \cdot 30} = 150$$

- It turns out, this function belongs to a common family of functions called **Cobb-Douglas Production Functions** which satisfy all three of our assumptions
- We can see what each assumption looks like in practice

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Assumption 1: Increasing Production in Each Input

- Let's check that F is increasing in N

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\frac{\partial F}{\partial N} &= \frac{\partial}{\partial N}\left[5\sqrt{K \cdot N}\right] \\
&= \frac{\partial}{\partial N}\left[5K^{\frac{1}{2}}N^{\frac{1}{2}}\right] \\
&= \left(\frac{5}{2}\right) K^{\frac{1}{2}} N^{-\frac{1}{2}} \\
&= \left(\frac{5}{2}\right)\left(\frac{K}{N}\right)^{\frac{1}{2}} \\
&> 0
\end{aligned}
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so long as K and N are positive

The argument is exactly the same for K

- Note: $\partial F/\partial N$ is *exactly* the marginal product of labor (MPL)

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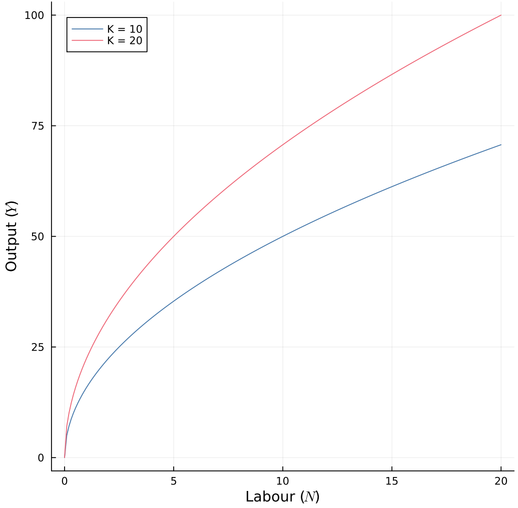

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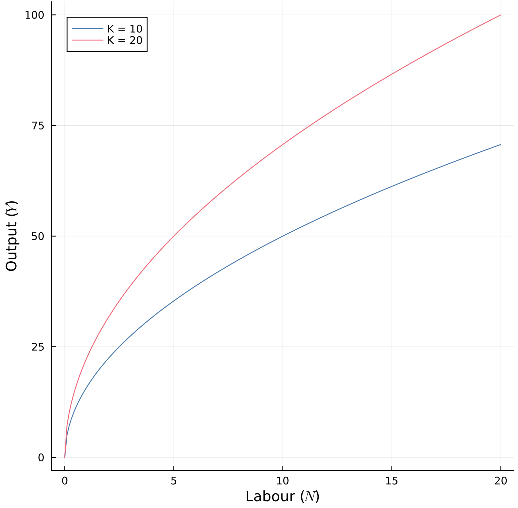

Assumption 2: Diminishing marginal products

- Let's plot the marginal product of labor
- We can check it mathematically as well

$$\begin{aligned}\frac{\partial^2 F}{\partial N} &= \frac{\partial}{\partial N}\left[MPL(K,N)\right] \\ &= \frac{\partial}{\partial N}\left[\left(\frac{5}{2}\right)K^{\frac{1}{2}}N^{-\frac{1}{2}}\right] \\ &= -\left(\frac{1}{2}\right)\left(\frac{5}{2}\right)K^{\frac{1}{2}}N^{-\frac{3}{2}} \\ &< 0\end{aligned}$$

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- Each worker adds something, but their marginal contribution becomes smaller
- The workers are sharing the same fixed capital stock among as the number of workers increases

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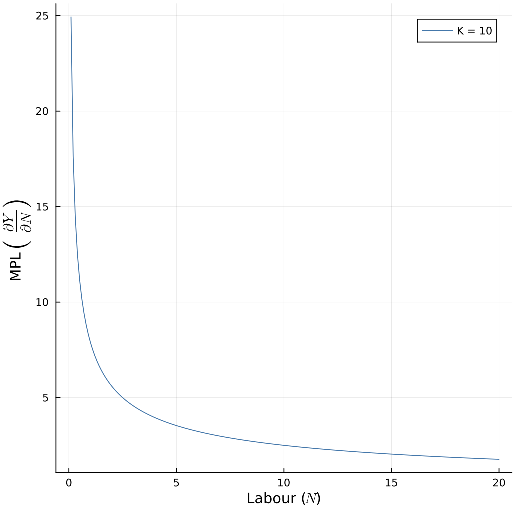

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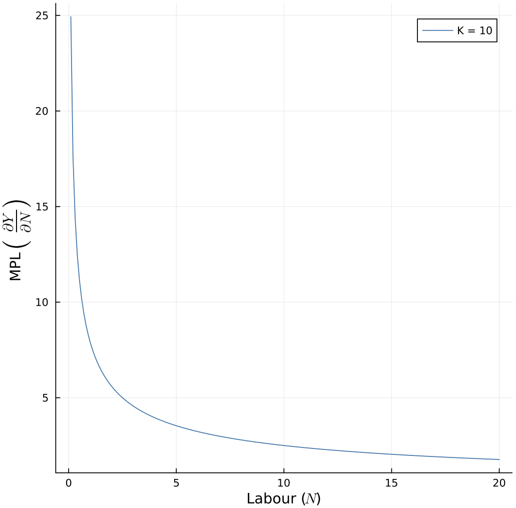

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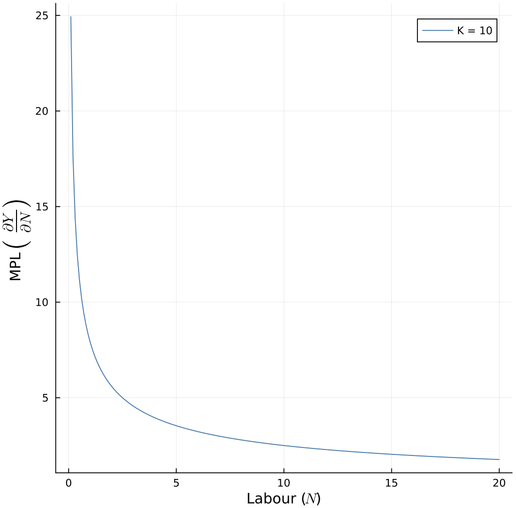

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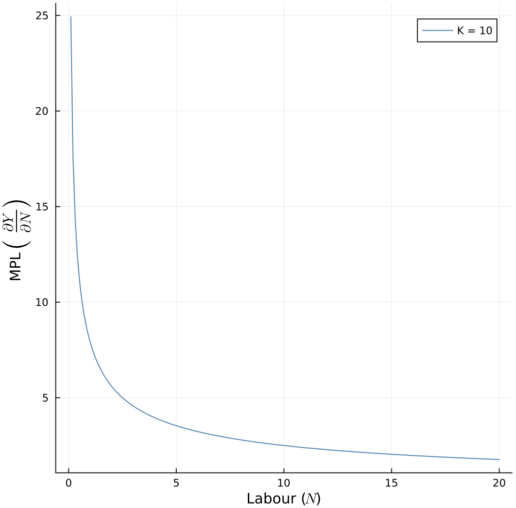

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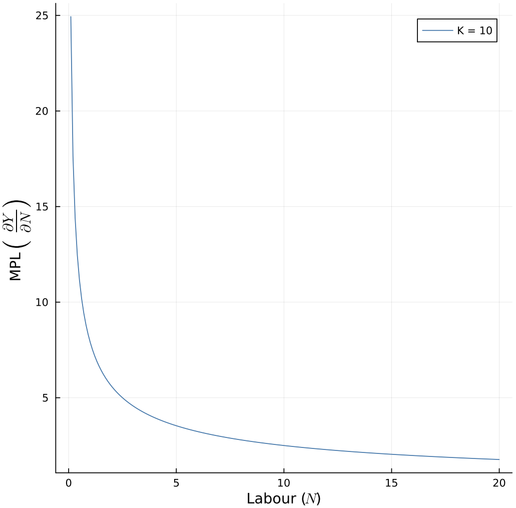

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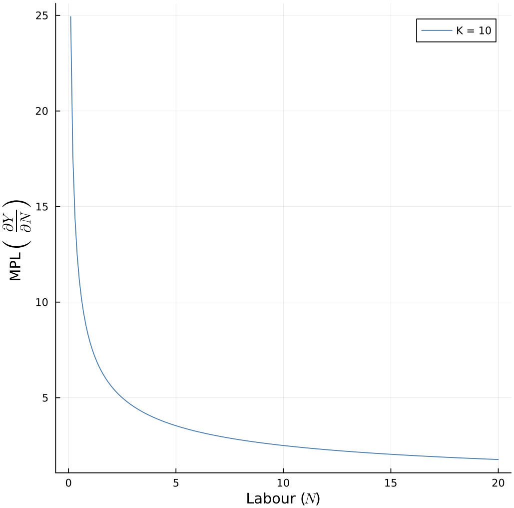

Assumption 3: Constant Returns to Scale

- Recall that CRS means that for any scaling factor λ,

$$F(\lambda K, \lambda N) = \lambda F(K, N)$$

I.e, Doubling the inputs doubles the outputs

- Let's check this for our simple example:

$$\begin{aligned} F(K, N) &= 5\sqrt{K \cdot N} \\ F(\lambda K, \lambda N) &= 5\sqrt{\lambda^2 K \cdot N} = \lambda 5\sqrt{K \cdot N} \\ &= \lambda F(K, N) \end{aligned}$$

- Geometrically, if we plot the **isoquants** (level sets) of the production function, then...
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- Recall that CRS means that for any scaling factor λ,

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  I.e, Doubling the inputs doubles the outputs

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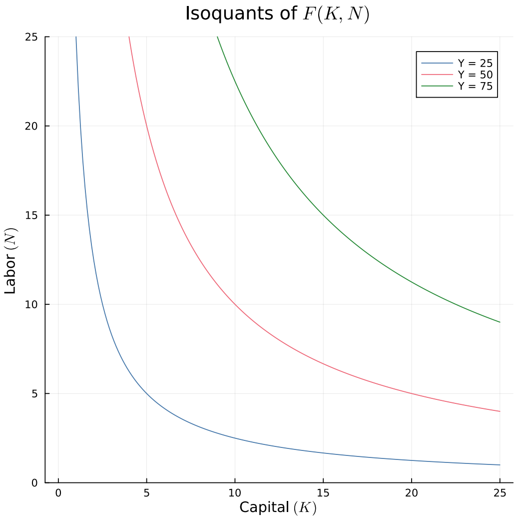

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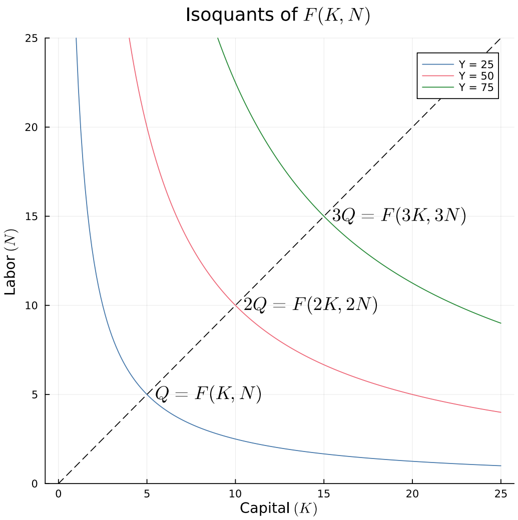

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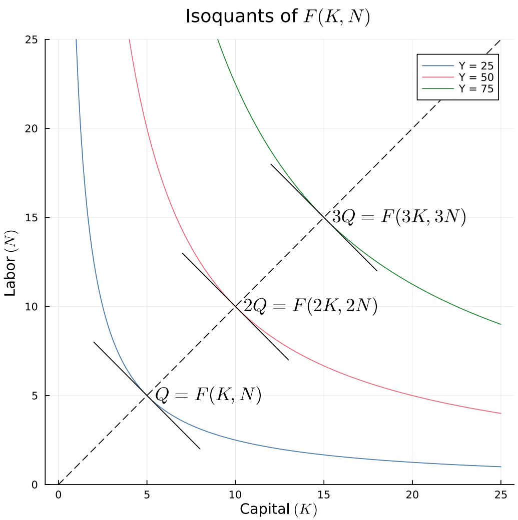

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- Let's think about the alternatives
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Cobb-Douglas Production Functions

- In macro, we often use a **Cobb-Douglas** production function

$$Y = K^{\alpha}(EN)^{1-\alpha}$$

  where E is the variable level of technology (how efficient is your labor)

  Note: to be consistent with Gottfries, we're treating E as being labor augmenting, but you should be able to see that it doesn't matter if we it this way, or in the more standard way

$$Y = \tilde{E} \times K^{\alpha} N^{1-\alpha}$$

  for an appropriate definition of $\tilde{E}$. This is the more standard way of writing the production function

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Cobb-Douglas has positive marginal products (Assumption 1)

Let's do this for capital. Recall that

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We can see then that:

$$\begin{aligned}
MPK &= \frac{\partial Y}{\partial K} && \text{By definition} \\
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Similar argument shows that

$$MPL = (1-\alpha)E\left(\frac{K}{EN}\right)^{\alpha} > 0 \tag{2}$$

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What determines the MPL?

- We're often interested in the MPL because it is a key determinant of the real wage
- So what drives it? Recall...

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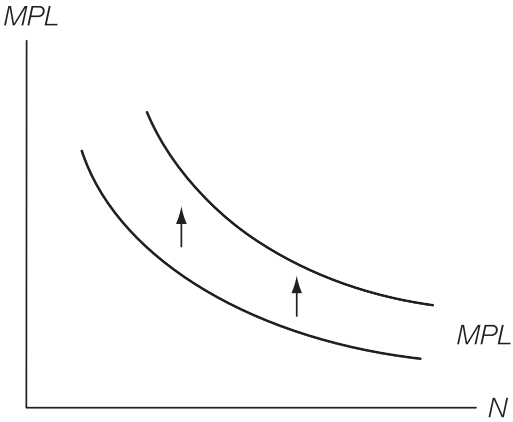

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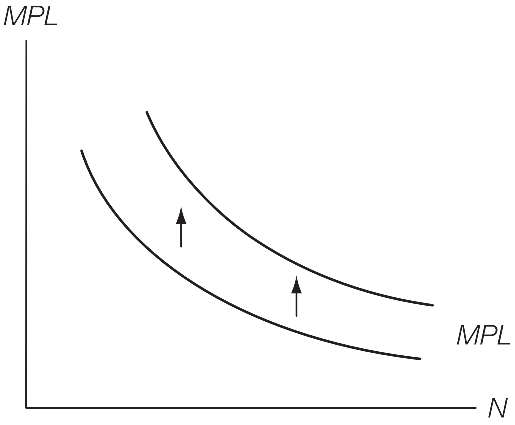

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As we mentioned last week, firms and consumers have simple goals

The goal of the firm is to maximize profits:

$$\text{Profits} = \text{Total revenue - total costs}$$

The firm decides how many units to produce and sell. Should it produce and sell one more unit? Will that lead to more profits?

Depends on:

- *Marginal revenue:* the extra revenue gained from producing one extra unit
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- *Marginal revenue:* the extra revenue gained from producing one extra unit
- *Marginal cost:* the extra cost incurred from producing one extra unit

The firm will produce until marginal revenue equals marginal cost!

Profit maximization–Intuition

What if $MR > MC$?
The revenue we make from producing one extra unit is greater than the cost we incur of that one extra unit. But then we can produce more and make more profit!!

What if $MR < MC$?
The revenue we make from producing the last unit is smaller than the cost we incurred from producing it. But then we should not have produced that last unit? We can make more profit by producing less!!

What if $MR = MC$?
At this point, the firm can increase revenue by producing more, but the cost of doing that is equal, so that is not tempting. It can also reduce the cost by producing less, but the loss in revenue is equal, so that is not tempting either. The firm is at its profit maximizing point!

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Profit maximization

- Suppose we have a cost function $C(Y_i)$ and a revenue function $R(Y_i)$
- The firm needs to decide how much output Y_i to produce, so they solve the problem

$$\max_{Y_i} R(Y_i) - C(Y_i)$$

- Taking first order conditions (set derivative of the objective function equal to zero), we know that firm optimality requires:

$$R'(Y_i) - C'(Y_i) = 0$$
$$\implies \underbrace{R'(Y_i)}_{\text{Marginal Revenue}} = \underbrace{C'(Y_i)}_{\text{Marginal Cost}}$$

- The firm produces where the marginal revenue is equal to the marginal cost in order to maximize profits!
- We're now going to spend some time really thinking through the relationship between output and revenues, which is going to depend, crucially, on the assumptions we make about the market structure of economy.

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Monopolistic Competition

- There are a large number of firms
- Each firm produces a good that is *similar but not identical* to the goods of the other firms, in other words, the firms produce **imperfect substitutes**
- This means that each firm has *some* "monopoly power"
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Market power under monopolistic competition

Consider the perspective of one specific firm (call them firm i). Demand for firm i's good depends on:

- The price the firm charges for its products (P_i)
- How much other firms are charging for their products (aggregate price level P)
- Total demand in the economy (Y)

In particular, we're going to make a specific assumption about the functional form of their demand:

$$Y_i = D\left(\frac{P_i}{P}\right) \cdot Y$$

for some demand function D. Note: this doesn't come out of thin air. This functional form can be derived from a Dixit-Stiglitz demand system (which you'll learn about more next year)

- We can rewrite this as:

$$\frac{Y_i}{Y} = D\left(\frac{P_i}{P}\right)$$

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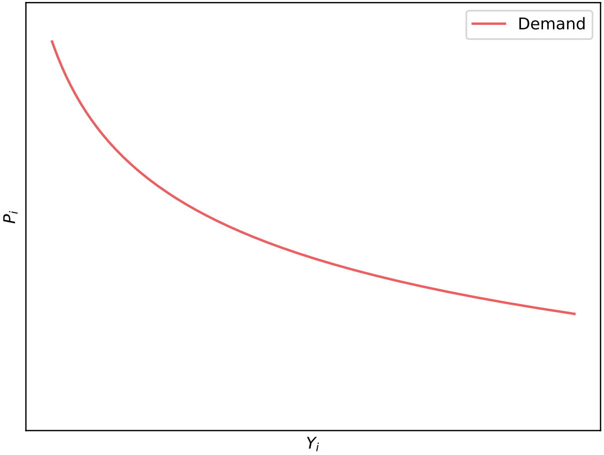
Downward sloping demand curve

- For any fixed aggregate price P and fixed Y, a higher P_i leads to a lower Y_i
- Note: we've drawn this curve smoothly, rather than linearly
- We usually want to think about **iso-elastic** demand curves (since they tend to be very nice and easy to work with)



Price Elasticity of Demand

- The degree of competition in the market typically depends on how **elastic** demand is
- How much does the firm lose in sales if it increases its price? (in percentage terms)
- The closer the substitutes that are available, the less monopoly power the firm has (i.e, the harder it is, in terms of sales, for the firm to raise its prices)
- We will denote by η the **price elasticity of demand**:

$$\eta = \frac{dY_i/Y_i}{dP_i/P_i} = \frac{dY_i}{dP_i}\frac{P_i}{Y_i}$$

Note: I've implicitly assumed, by writing it as a single variable, that η is constant. This is because we're assuming that firm demand is **iso-elastic**. I.e, it has a constant elasticity no matter the level of demand. Again, this comes from the Dixit-Stiglitz demand system.

- In monopolistic competition, we will assume that

$$-\infty < \eta < -1$$

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Price elasticity of demand

- **Why is η negative?**

  If the price falls, demand increases. (and vice versa)

- **Why is $\eta > -\infty$?**

  In a perfectly competitive market, we would have $\eta = -\infty$, because sales fall to zero if the price is raised (infinitesimally) above market price. Our firms will have some ability to raise the price and still sell something.

- **Why is $\eta < -1$?**

  If we had $\eta > -1$ a 1 percent increase in price leads to a loss of less than one percent in goods sold, i.e. an increase in revenue. Moreover, since the amount produced falls, the total production cost also goes down. Thus, the firm could just keep on increasing their prices and make more and more profit. Doesn't make economic sense.

Inverting the demand curve

- Instead of viewing the level of output as a function of price:

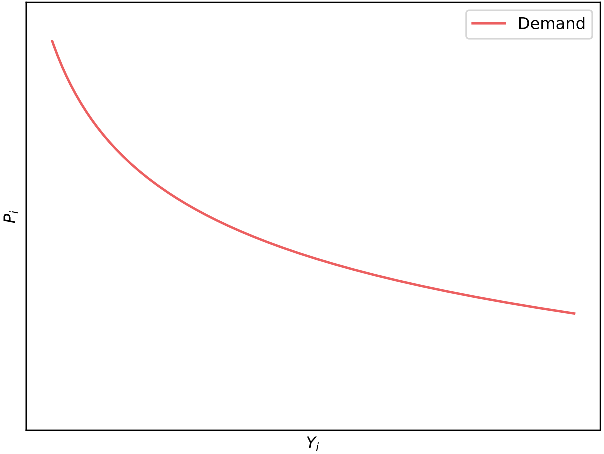
$$Y_i = D(P_i)$$

we can view the price as a function of how much the firm plans to sell:

$$P_i = P_i(Y_i)$$

where P_i is the price the firm has to charge.

- This function is called the **inverse demand function**.



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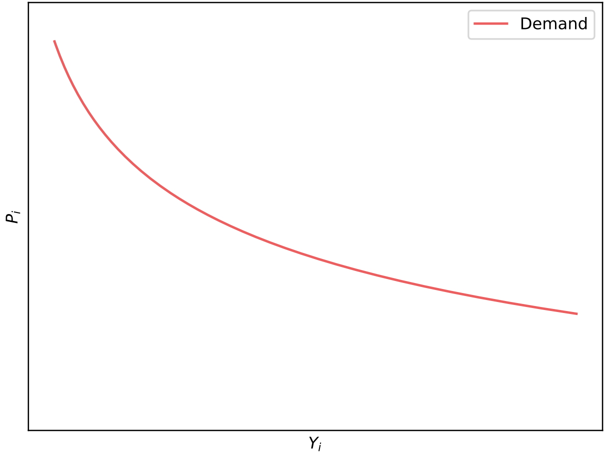

Total revenue and marginal revenue

- Total revenue is all the money that the firm sells it goods for. Thus, total revenue:

$$R(Y_i) = Y_i \cdot P_i = Y_i \cdot P_i(Y_i)$$

- How much does total revenue change if you sell one additional unit? Take the derivative with respect to Y_i!

$$\underbrace{R'(Y_i)}_{\text{Marginal Revenue}} = \underbrace{P_i(Y_i)}_{\substack{\textit{Revenue from selling} \\ \textit{one more unit}}} + \underbrace{Y_i \cdot \frac{dP_i}{dY_i}}_{\substack{\textit{Decrease in revenue from} \\ \textit{lower price on all units sold}}} \qquad \text{Using the product rule}$$

$$= P_i(Y_i) + Y_i \cdot \frac{dP_i}{dY_i} \cdot \frac{P_i(Y_i)}{P_i(Y_i)} \qquad \text{Multiply and divide by } P_i(Y_i)$$

$$= \left(1 + \frac{dP_i}{dY_i} \cdot \frac{Y_i}{P_i(Y_i)}\right) P_i(Y_i) \qquad \text{Collect terms}$$

- From here on, we're going to drop the notation $P_i(Y_i)$ and just write P_i, with the implicit understanding that choosing Y_i and choosing P_i are equivalent

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Marginal revenue

- So we now have:

$$R'(Y_i) = \left(1 + \frac{dP_i}{dY_i} \cdot \frac{Y_i}{P_i}\right) P_i$$

This should, this looks vaguely familiar...

- Remember the expression for the price elasticity of demand:

$$\eta = \frac{dY_i}{dP_i} \frac{P_i}{Y_i}$$

- It turns out that if you have an iso-elastic demand function, the output elasticity of the inverse demand function is just $\frac{1}{\eta}$.

This is not obvious, and requires some work to show. You'll be showing this in Q8 on the tutorial sheet.

- Thus, we get:

$$R'(Y_i) = \underbrace{\left(1 + \frac{1}{\eta}\right)}_{<1} P_i$$

Remember that $-\infty < \eta < -1$, thus $-1 < 1/\eta < 0$

Marginal revenue

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$$\eta = \frac{dY_i}{dP_i}\frac{P_i}{Y_i}$$
- It turns out that if you have an iso-elastic demand function, the output elasticity of the inverse demand function is just $\frac{1}{\eta}$.
This is not obvious, and requires some work to show. You'll be showing this in Q8 on the tutorial sheet.
- Thus, we get:
$$R'(Y_i) = \underbrace{\left(1 + \frac{1}{\eta}\right)}_{<1} P_i$$

Remember that $-\infty < \eta < -1$, thus $-1 < 1/\eta < 0$

Marginal revenue

- So we now have:

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Marginal revenue is positive, but smaller than the price.

Suppose a firm:

- ...sells 100 units
- ...and faces a price elasticity of demand of -2
- What happens if this firm wants to sell one more unit? (wants to increase the output by one percent)

To be able to sell one more unit, it has to decrease its price. By how much? $1/2$ percent!

So how did this increase in sales by one unit (i.e. one percent) change the revenue of the firm?

$$\underbrace{P_i}_{\substack{\text{Revenue from the}\\ \text{extra unit sold}}} - \underbrace{0.005 \cdot 100 \cdot P_i}_{\substack{\text{Loss from lowering}\\ \text{the price of all units sold}}} = (1 - 0.5)P_i = 0.5P_i$$

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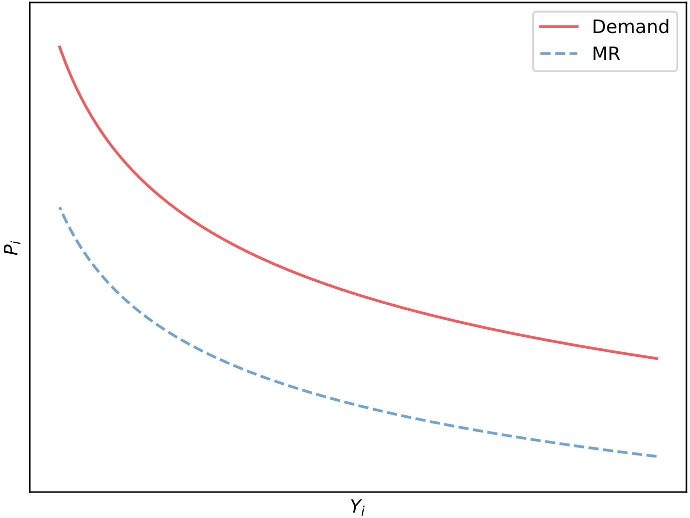
Demand curve and marginal revenue



Wrapping up

- The production function is a representation of the aggregate technology–it tells us how much can be produced with a given input
  - One particularly useful function is the **Cobb-Douglas**
- A firm's goal is simple: it wants to maximize profits
  - In general achieved by setting MR = MC
- How is profit maximization achieved in our particular model where we assume monopolistic competition?
  - We analyzed the firm's demand curve…
  - …and figured out how much the firm's marginal revenue changes if they sell one additional unit of good

Finally, we will spend some time figuring out how the firm sets its price!

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Taking stock

We have answered the following questions about a firm:

- What is the firm's goal? *(profit maximization!)*
- How does the firm achieve its goal–in general? *(MR=MC)*

Then we started figuring out how profit maximization is achieved in our particular model:

- Which market structure do we assume? *(monopolistic comp.)*
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- How much does the firm's marginal revenue change if they sell one additional unit of good? *(less than the price)*

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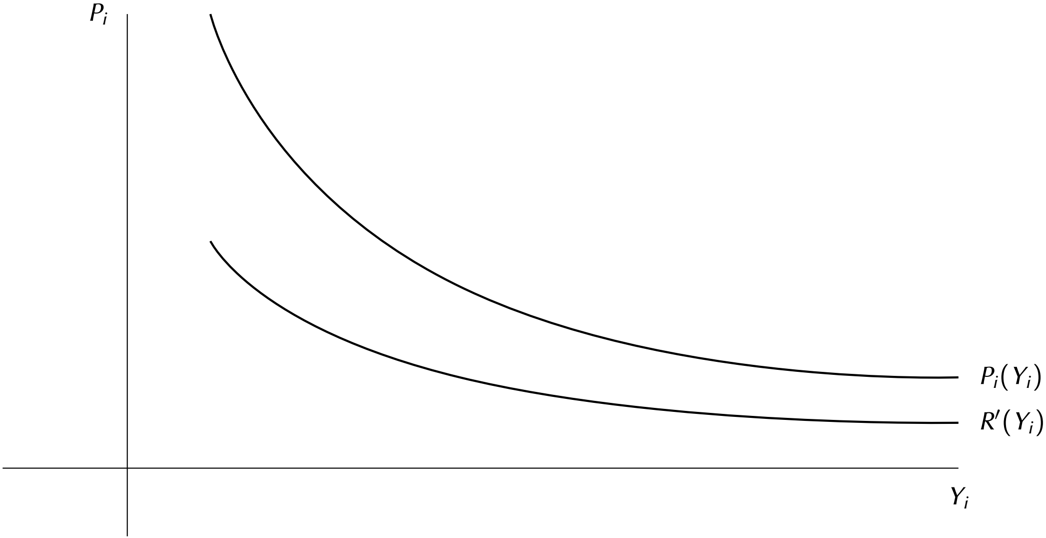
Prices and Markups



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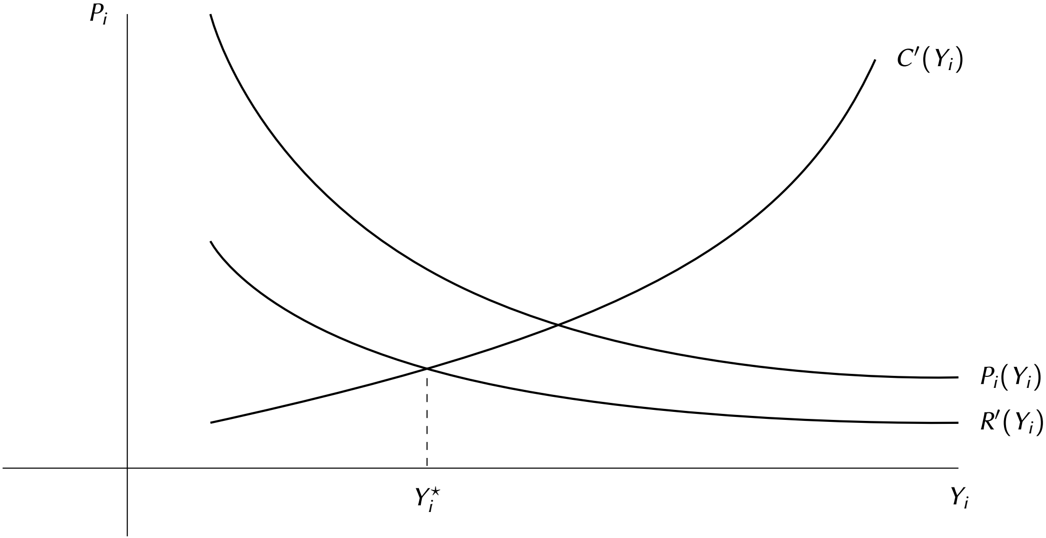

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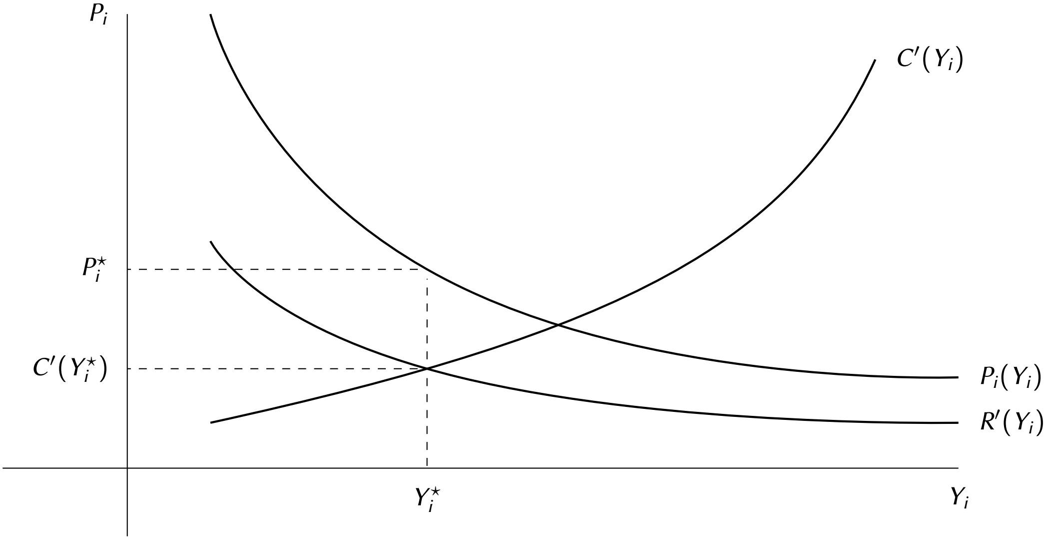

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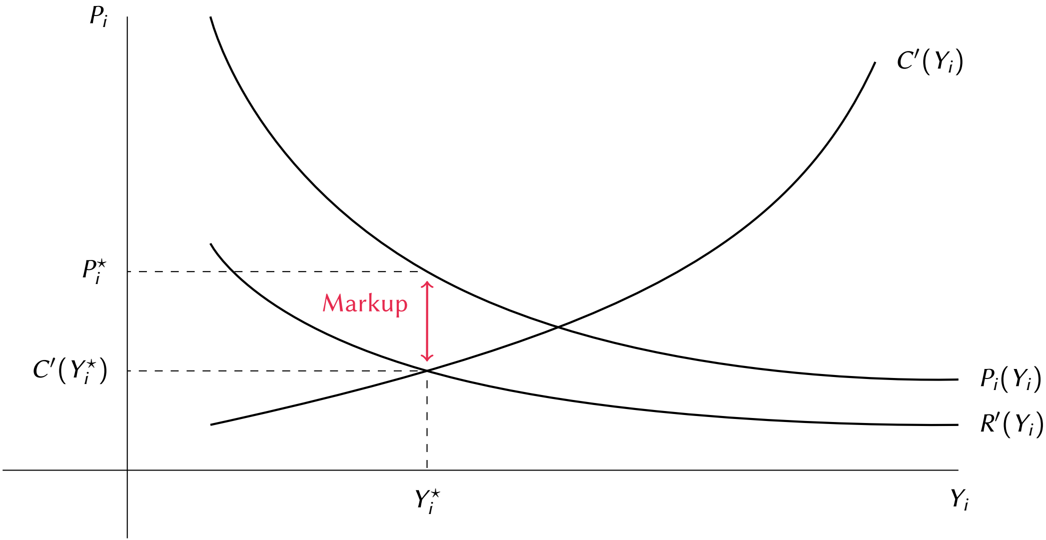

Mark-up: an example

With a price elasticity of demand (PED) of $\eta = -5$, we get

$$1 + \mu = \frac{1}{1 + \frac{1}{-5}} = \frac{1}{0.8} = 1.25 \quad \Rightarrow \quad \text{mark-up is } 25\%$$

With a price elasticity of demand (PED) of $\eta = -2$, we get

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With a price elasticity of demand (PED) of $\eta = -11$, we get

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Mark-up and price elasticity of demand

Let's verify if these results make economic sense.

$\eta = -2$

- If a firm has a price elasticity of demand of -2, it means that if they increase their price 1%, the quantity they sell falls by 2%
- Thus, it is "not very costly" to charge a higher price
- The smartest price will be pretty high

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Mark-ups and monopolistic competition

The mark-ups we see is an effect of the monopolistic competition, and reflect the degree of competition

- The $\eta = -11$ in our example
  - Is the more competitive market
  - Is where the goods are better substitutes
  - Has a lower mark-up
- We will take the level of competition, and thus the level of mark-ups, as given *exogenously*
- Nevertheless, we will be able to analyze how the degree of competition affects the economy

$$P_i = (1 + \mu)C'(Y_i)$$

To close this analysis, we will need to say something about how marginal costs are determined

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Marginal cost

The marginal cost (MC) measures how much costs increase if a company produces one more unit

First of all, we have to figure out how we increase production

- For now, we are going to assume that the capital stock is given, and that we cannot adjust it
- However, what we can adjust is the amount of labour
- So to produce more, we need to hire more workers (or more hours worked by our workers)

Next, we have to figure out how much this costs us–per unit of goods produced!

The Firm's Cost Function

- Suppose that the firm wants to produce an output of Y_i. How much will this cost them?
- Recall that:

$$Y_i = F(K_i, N_i)$$

  What variables do they get to choose?
  - N_i: Labor can be adjusted freely
  - K_i: They can't choose the capital stock today (it was determined last period by their investment decisions)
- Firm will choose labor to produce Y_i at the lowest cost possible

  Remember that cost minimization is the same as profit maximization
- Let's write the firm's problem, taking K_i as a given

$$C(Y_i) = \min_{N_i} \quad W_i \cdot N_i$$
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Deriving the marginal cost

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- The constraint implies that N_i is a function $N_i(Y_i)$ since F is strictly increasing in N_i
- Now, let's take the derivative of the constraint with respect to Y_i:

$$1 = F_K(K_i, N_i) \underbrace{\frac{\partial K_i}{\partial Y_i}}_{=0} + F_N(K_i, N_i) \frac{\partial N_i}{\partial Y_i} = F_N(K_i, N_i) \frac{\partial N_i}{\partial Y_i}$$

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- This implies that

$$\frac{\partial N_i}{\partial Y_i} = \frac{1}{F_N(K_i, N_i)} = \frac{1}{\text{MPL}}$$

- Finally, we can plug all this back into the original problem to find that:

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Deriving the marginal cost

$$C(Y_i) = \min_{N_i} \quad W_i \cdot N_i$$
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Marginal cost: Intuition

- How much does the extra worker we hire produce?
  - Depends on how productive that extra worker is
- How do we measure the productivity of one additional (marginal) worker?
  - By the MPL, the marginal product of labour!

Assume that the MPL is 5 units (one additional worker produces 5 units of goods)

How much do we have to pay this worker? The wage. Say it is £10.

What is then the cost for producing *one* more unit? (£/unit)

$$MC = \frac{1}{5} \cdot 10 = 2$$

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Marginal cost and market price charged

- Now we can finally go back to the price the firm will charge!
- We showed that to maximize profits, firm i will choose the price P_i such that:

$$P_i = (1+\mu)C'(Y_i)$$

- But now we know what the marginal cost is, so we get, for every individual firm i

$$P_i = (1+\mu)\frac{W_i}{\text{MPL}_i}$$

- How do we go from the individual firm level back up to the aggregate price?
- Recall our assumption: **All firms are identical!** So that means that if this is the solution for one firm, it is the solution for all firms

$$P = (1+\mu)\frac{W}{\text{MPL}}$$

This aggregation is tricky – it relies on the fact that we have CRS, so $\text{MPL}_i = \text{MPL}$

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Understanding marginal cost for our Cobb-Douglas case

Remember from that we often use the Cobb-Douglas production function:

$$Y = K^{\alpha}(EN)^{1-\alpha}, \quad 0 < \alpha < 1; \quad K, E, N > 0$$

$$\Rightarrow MPL = (1-\alpha)K^{\alpha}E^{1-\alpha}N^{-\alpha}$$

MPL: measures how much production increases per additional worker

How does MPL behave as N, K, or E vary?

- The more N the lower is MPL
- The more K the higher is MPL
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+ mark-ups
+ wage level
- better technology (lowers the marginal cost)
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Summarizing price setting

We have now finally figured out how the firm sets its price

- The firm maximizes profits by setting $MR = MC$
- With a monopolistic competition market structure, the marginal revenue is a fraction of the price:

$$MR = \left(1 + \frac{1}{\eta}\right) P_i$$

- Given a fixed amount of capital, the marginal cost is

$$MC = W/MPL$$

- Thus, putting it all together:

$$P = (1 + \mu)\frac{W}{MPL} \qquad \text{where } 1 + \mu := \frac{1}{1 + \frac{1}{\eta}}$$

- And if we assume a Cobb-Douglas production function (which we do!) we can be even more specific

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The natural level of production

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- Last few lectures we have been thinking as a firm, figuring out how the production function works, and how the firm sets its price
- Now it is time to consider what determines production in the economy as a whole
- Production in the economy is determined by:
  - The amount of capital (K)
  - The amount of workers (N)
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- At any one point in time, K and E are given (determined by historical actions that we will analyze in Block 2)
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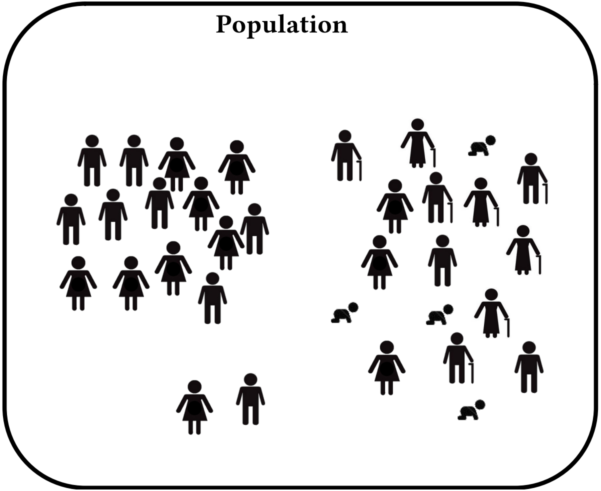
What determines N?

L The labor force (which we take as given)

u The unemployment level (U/L)

N Number of workers, employment



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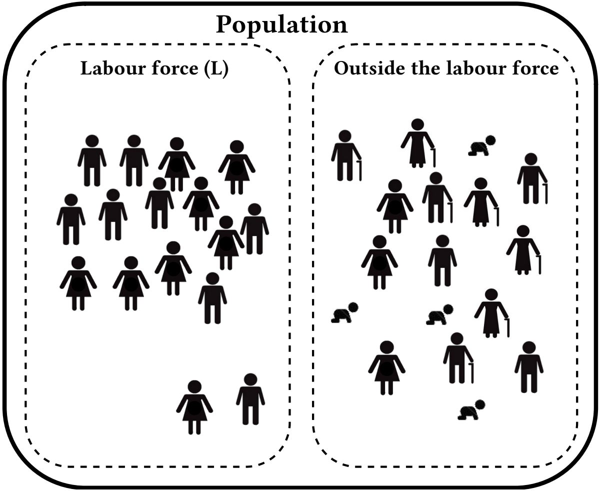

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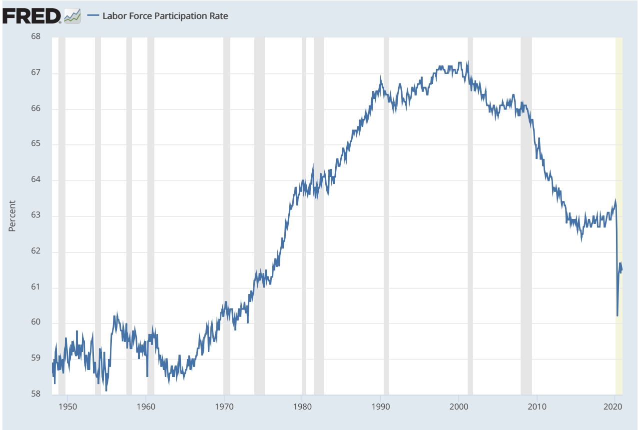

Figure: US Labor Force Participation since 1950 (+16)

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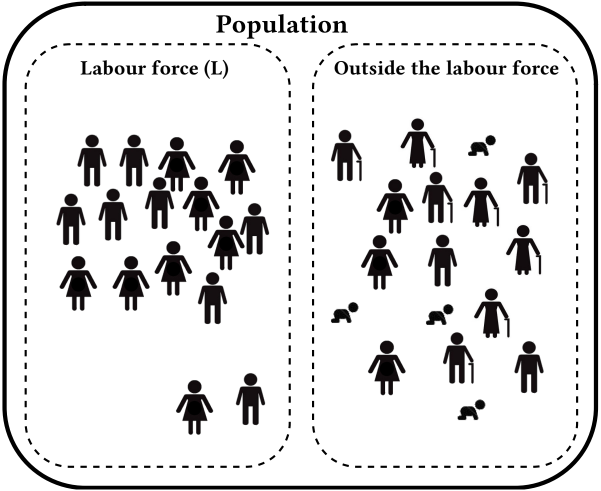

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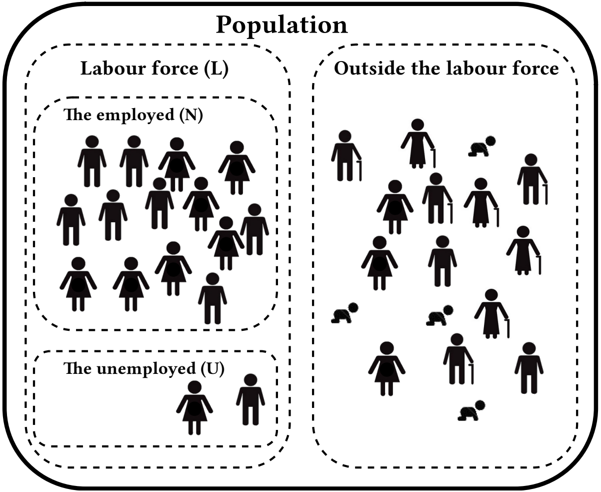
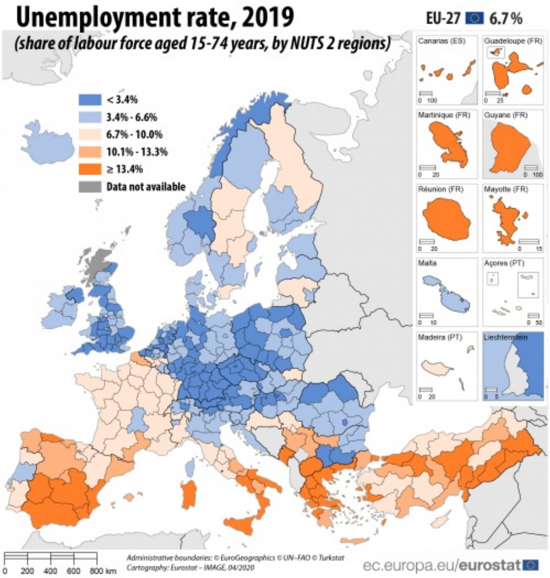


Unemployment across space



Unemployment across time

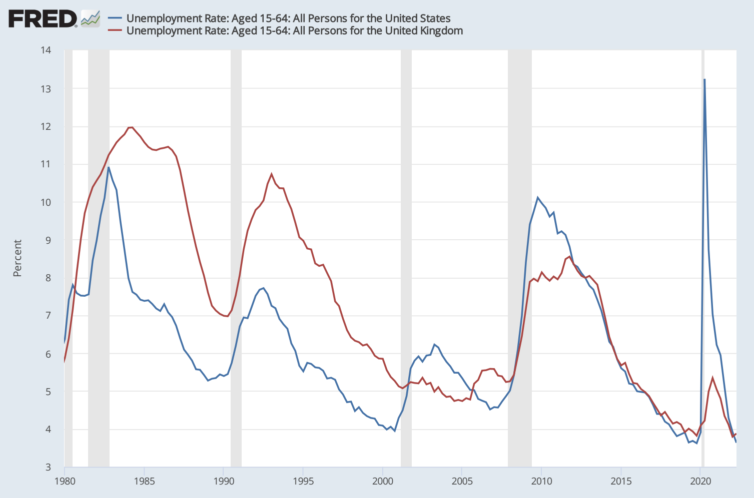


Figure: Source: https://fred.stlouisfed.org/

Even in good times there is some unemployment

Some reasons for ongoing unemployment:

- Search frictions
- Unemployment insurance

Natural rate of unemployment: u^n

- Later in the course, you will discuss that there is some "normal" or "long-run equilibrium" level of unemployment
- For now, we just take it as given
- Loosely speaking: on average, unemployment is at this level

Natural level of production

Production when $N = (1 - u^n)L = N^n$
(given the capital stock and the technology level):

$$Y^n = F(K, E(1 - u^n)L)$$

The **natural level of production** today depends on:

- K Amount of capital (+)
- E Technology level (+)
- L Size of labour force (+)
- u^n Natural rate of unemployment (-)

Natural level of production

You will later in the course discuss what could change the natural level of production in the longer run:

- Growth in technology and capital
- Changes in u^n (can be affected by policy!)

In the shorter run, we take the natural level as given, *but unemployment and production might deviate* from their natural levels.

- The natural level of production is the reference point around which the economy fluctuates
- Helps us analyze business cycles!

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Labour's share of income

Now we have figured out production, and its "natural level"

- In a closed economy, production = income
  (remember Week 1?)
- Who gets the income?
- We have three types of income:
  - Labour income (wages)
  - Capital income (returns to the owners of buildings, machinery, etc)
  - Pure profits (economic rents to the owners of the monopoly power)
- Let us start thinking about the labour income

The simplest case

First, think about the absolutely simplest case: a world with no markups, no difference between real and nominal, and a Cobb-Douglas production function:

$$Y = K^{\alpha} N^{1-\alpha}$$

In this simple world, the (real) wage is equal to the marginal product of labour, MPL .

$$W = MPL = (1-\alpha) K^{\alpha} N^{-\alpha}$$

How large is the total wage bill?

$$WN = (1-\alpha) K^{\alpha} N^{-\alpha} N = (1-\alpha) Y$$

i.e. a constant fraction of output!

The simplest case

How large is the capital share of income? It must be the remaining part of GDP:

$$\textit{Capital income } = Y - (1-\alpha)Y = \alpha Y$$

Or, we can think about it as the rental rate of capita (the remuneration or payment to the capital) times the amount of capital:

$$MPK = \alpha K^{\alpha-1} N^{1-\alpha}$$
$$MPK \cdot K = \alpha K^{\alpha-1} N^{1-\alpha} K = \alpha Y$$

Our slightly more complicated case

In the simplest case, we concluded that α is the key parameter that decides the labour share of income and the capital share of income.

- α Measures the importance of capital vs. labour in the production function (elasticity of output with respect to capital)

What is the difference in our case?

- We have monopolistic competition, so we have markups
- We have a price level (difference between nominal and real)

Let's see what happens in our model!

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The real wage

Workers don't (or shouldn't) care about the nominal wage. What is important is the **real wage**: the wage expressed in purchasing power, i.e. how much you can buy for your wage.

We have already figured out the price equation:

$$P = (1 + \mu)\frac{W}{MPL}$$

Rearranging this, we immediately see the real wage:

$$\frac{W}{P} = \frac{MPL}{1 + \mu}$$

Some quick observations about the real wage

$$\frac{W}{P} = \frac{MPL}{1+\mu}$$

- Increasing in MPL, decreasing in markups
- If we had zero markups (i.e. perfect competition) it would have been exactly MPL

Remember that **with Cobb-Douglas production** we have $MPL = (1-\alpha)Y/N$:

$$\frac{W}{P} = \frac{1-\alpha}{1+\mu} \cdot \frac{Y}{N}$$

The real wage is a (constant) fraction of GDP per worker!

Distribution of income

$$\frac{W}{P} = \frac{1-\alpha}{1+\mu} \cdot \frac{Y}{N}$$

We can rearrange this:

$$\frac{WN}{PY} = \frac{1-\alpha}{1+\mu}$$

WN Total (nominal) labour income

PY Total (nominal) production

Thus, we have found the **labour share of income**!

We can also think about it as $\frac{(WN)/P}{Y}$, the *real* total wage bill as a share of *real* GDP.

Labour share of income

$$\frac{WN}{PY} = \frac{1-\alpha}{1+\mu}$$

What does the **labour share** of income depend on?

- α The importance of capital in the production process
- μ The level of markups (which depends on the level of competition)

What does it **not** depend on?

- K The amount of capital
- N The amount of labour
- E The current level of technology

We normally think that both α and μ are relatively stable over time and relatively similar across countries.

Then the labour share of income should be constant??

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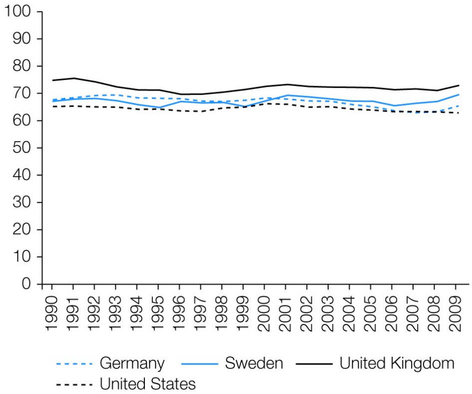
Labour share of income

Fig. 2.9 *The labour share in the United Kingdom, United States, Germany, and Sweden*



Source: The Conference Board Total Economy Database, January 2011, http://www.conference-board.org/data/economydatabase/.

Figure: Source: Gottfries textbook

One reason why we like the Cobb-Douglas production function!

Summing up

This week we have figured out the supply side of the economy:

- How much can be produced?
- What determines the prices? (P and W)

Remember Week 1: we can view GDP from three perspectives:

- ✓ Production side
- ✓ Income side
- User side

Next week, we will turn to the demand side (user side): who uses the goods produced?

end for today